Financial Fragility with SAM?

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Abstract

Shared Appreciation Mortgages feature mortgage payments that adjust with house prices. They are designed to stave off borrower default by providing payment relief when house prices fall. Some argue that SAMs may help prevent the next foreclosure crisis. However, the home owners' gains from payment relief are the mortgage lenders' losses. A general equilibrium model where financial intermediaries channel savings from saver to borrower households shows that indexation of mortgage payments to aggregate house prices increases financial fragility, reduces risk-sharing, and leads to expensive financial sector bailouts. In contrast, indexation to local house prices reduces financial fragility and improves risk-sharing.

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1 Introduction

The $10 trillion market in U.S. mortgage debt is the world's largest consumer debt market and its second largest fixed income market. Mortgages are not only the largest liability for U.S. households, they are also the largest asset of the U.S. financial sector.[1] Given the heavy exposure of the financial sector to mortgages, large house price declines and the default waves that accompany them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the Great Financial Crisis of 2008-2011, as U.S. house prices fell by 30% nationwide, and by much more in some regions, pushing roughly 25% of U.S. home owners underwater by 2010, and leading to seven million foreclosures. Large losses on real estate loans caused several U.S. banks to collapse during the crisis, while the stress to surviving banks' balance sheets led them to dramatically tighten mortgage lending standards, precluding many home owners from refinancing into lower interest rates.[2] Homeowners' reduced ability to tap into their housing wealth short-circuited the stimulative consumption response from lower mortgage rates that policy makers had hoped for.

This experience led economists and policy makers to ask whether a different mortgage finance system would result in a better risk sharing arrangement between borrowers and lenders.[3] While contracts offering alternative allocations of interest rate risk are already widely available — most notably, the adjustable rate mortgage (ARM), which offers nearly perfect pass-through of interest rates — contracts offering alternative divisions of *house price* risk are still rare. Recently, however, some fintech lenders have begun to offer such contracts — most notably the shared appreciation mortgage (SAM), which indexes mortgage payments to house price changes.[4]

A SAM contract ensures that the borrower receives payment relief in bad states of the world, potentially reducing mortgage defaults and the associated deadweight losses to society. However, SAMs impose losses on mortgage lenders in these adverse aggregate states, which may increase financial fragility at inopportune times. Our paper is the first to study

[1] Banks and credit unions hold $3 trillion in mortgage loans directly on their balance sheets in the form of whole loans, and an additional $2.2 trillion in the form of mortgage-backed securities.Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITs in the definition of the financial sector adds another $1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and the GSE portfolios adds a further $2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

[2] Charge-off rates of residential real estate loans at U.S. banks went from 0.1% in mid-2006 to 2.8% in mid-2009, returning to their initial value only in mid-2016.

[3] The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015.

[4] Examples of startups in this space are Unison Home Ownership Investors, Point Digital Finance, Own Home Finance, and Patch Homes. In addition, similar contracts have been offered to faculty at Stanford University for leasehold purchases over the past fifteen years (Landvoigt, Piazzesi, and Schneider, 2014).

how SAM contracts affect the allocation of house price risk between mortgage borrowers, financial intermediaries, and savers in a general equilibrium framework. It proposes a shift in the mortgage design literature from a focus on *household risk management* to one on *system-wide risk management*. The main goal of this paper is to quantitatively assess whether SAMs present a better arrangement to the overall economy than standard fixed-rate mortgages (FRMs).

We begin with a rich baseline model where mortgage borrowers obtain long-term, defaultable, prepayable, nominal mortgages from financial intermediaries. These intermediaries are financed with short-term deposits raised from savers and equity raised from their shareholders, subject to realistic capital requirements, and are bailed out by the government in case of insolvency. Borrowers face idiosyncratic house valuation shocks while banks face idiosyncratic profit shocks, which influence their respective optimal default decisions. We solve the model using a state-of-the-art global non-linear solution technique that allows for occasionally binding constraints.

To evaluate the mortgage system's resilience to adverse scenarios, our model economy transits between a normal state and a crisis state featuring high house price uncertainty and a fall in aggregate home values, in addition to aggregate business-cycle income risk. With FRMs, the arrival of a crisis state leads to higher rates of borrower defaults, bank losses and failures, along with large falls in borrower consumption as the financial sector contracts.

To study the impact of alternative mortgage contracts, we consider SAM economies where mortgage payments are either indexed to aggregate house prices or to local house prices. We contrast the effects of alternative schemes on the model's key externalities: the deadweight losses and risk-sharing consequences of borrower and bank default. Our main result is that indexation to aggregate (national) house prices reduces borrower welfare even though it slightly reduces mortgage defaults, due to a severe increase in financial fragility. These contracts lead mortgage lenders to absorb aggregate house price declines, causing a wave of bank failures and triggering bailouts ultimately funded by taxpayers, including the borrowers. Equilibrium house prices are lower and fall more in crises with aggregate indexation. Ironically, intermediary welfare increases as they enjoy large gains from increased mortgage payments in housing expansions, and can charge higher mortgage spreads in a riskier financial system. Aggregate indexation is bad for overall welfare.

In sharp contrast, indexation of mortgage payments to the local component of house price risk only can eliminate up to half of mortgage defaults while reducing systemic risk. Banks' geographically diversified portfolios of SAMs allow them to offset the cost of debt forgiveness in areas where house prices fall by collecting higher mortgage payments from areas where house prices rise. Lower mortgage defaults in turn substantially reduce bank failures and dampen fluctuations in intermediary net worth, stabilizing the financial system, and reducing deadweight losses. Banking becomes safer, but also less profitable, due to a fall in mortgage spreads and in the value of the bailout option. As a result, welfare of borrowers and savers rises, at the expense of bank owners. Overall welfare increases. The empirically relevant case, which we label regional indexation, combines aggregate and local indexation. It generates modest welfare benefits to the economy.

Our main model assumes strategic default. We also consider an alternative model of mortgage default where the vast majority of defaults are liquidity-driven, as suggested by Ganong and Noel (2019b). We find that the implications of indexation are largely unchanged because the two models imply similar dynamics of leverage and default.

Last, we examine the consequences of several alternative SAM implementations. Indexing interest payments only—which are fixed only until the next borrower mortgage transaction—has much weaker effects than indexing principal. Asymmetric indexation, which allows payments to fall but never to rise, dramatically decreases mortgage default rates, but does so by shrinking average household leverage rather than by improving risk sharing. It causes major financial fragility. In robustness analysis, we show that our results continue to hold when savers cannot hold any mortgage debt directly, when we vary the risk absorption capacity of the intermediaries, when indexation is partial, and when bank bailouts are financed with government debt rather than instantaneous taxation. Our results imply that macro-financial considerations should play an important role in the design of mortgage contracts indexed to house prices.

Literature Review. This paper contributes to the literature that studies innovative mortgage contracts. Shiller and Weiss (1999) are the first to discuss the idea of home equity insurance policies. SAMs were first discussed in detail by Caplin, Chan, Freeman, and Tracy (1997); Caplin, Carr, Pollock, and Tong (2007); Caplin, Cunningham, Engler, and Pollock (2008). They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but are also useful to prevent a mortgage crisis in the first place. More recently, Mian and Sufi (2014) have proposed a Shared Responsibility Mortgage (SRM), a mortgage whose payments fall when the local house price index goes down, and return to the initial payment upon recovery, while lenders receive a share of home value appreciation upon sale. They argue that foreclosure avoidance raises house prices, shares wealth losses more equitably between borrowers and lenders, and boosts borrower spending and aggregate consumption after house price falls. We build on this literature through our analysis of intermediary and financial risk, which interacts with the borrower balance sheet risk discussed in these works.

Kung (2015) studies the effect of the disappearance of non-agency mortgages for house prices, mortgage rates and default rates in an industrial organization model of the Los Angeles housing market. While not the emphasis of his work, he also evaluates the hypothetical introduction of SAMs in the 2003-07 period, finding that SAMs would have enjoyed substantial uptake, partially supplanting non-agency loans. However, SAMs would have further exacerbated the boom and would not have mitigated the bust. Piskorski and Tchistyi (2018) also study mortgage design in a risk neutral environment. They emphasize asymmetric information about home values between borrowers and lenders and derive the optimal mortgage contract. The latter takes the form of a Home Equity Insurance Mortgage that eliminates the strategic default option and insures borrowers' home equity. Relative to these papers, we provide a quantitative equilibrium model of the entire U.S. housing market, with risk averse lenders, and endogenously determined risk-free rate and mortgage risk premium. Our emphasis on imperfect risk sharing and financial fragility complements their approach.

Our paper is distinct in the quantitative literature on mortgage design since we study the endogenous interaction of contract design and intermediary risk-bearing capacity in general equilibrium. Guren, Krishnamurthy, and McQuade (2018) and Campbell, Clara, and Cocco (2018) investigate the interaction of ARM and FRM contracts with monetary policy. These authors focus on interest rate risk, contrasting e.g., adjustable-rate and fixed-rate mortgages, as well as novel contracts with various forms of optionality.[5] Both papers model a rich borrower risk profile that includes a life cycle and uninsurable idiosyncratic income risk. Perhaps because interest rate risk is easier for banks to hedge than house price risk, these papers abstract from implications for financial fragility and use exogenous lender SDFs to price loans. In contrast, our framework studies the impact on financial fragility of changing banks' contractual exposure to house price risk that is difficult to hedge. As a result, our model emphasizes a rich intermediation sector with capital requirements, bank failures, and bailouts, while featuring a much simpler borrower sector. We see these approaches as highly complementary.

More generally, our paper connects to the quantitative macro-housing literature, provid-

[5] Related work on contract schemes other than house price indexation include Piskorski and Tchistyi (2011), who study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract; Kalotay (2015), who considers automatically refinancing mortgages or ratchet mortgages (whose interest rate only adjusts down); and Eberly and Krishnamurthy (2014), who propose a mortgage contract that automatically refinances from a FRM into an ARM, even when the loan is underwater.

ing a novel and tractable general equilibrium setting for analyzing the interaction between the housing and financial sectors.[6] Our paper also contributes to the literature that studies the amplification of business cycle shocks provided by credit frictions, focusing specifically on key features of the mortgage market.[7] Finally, we provide a general equilibrium counterpart to recent empirical work that has found strong responses of consumption and default rates to changes in mortgage interest rates and house prices .[8]

Overview. The rest of the paper proceeds as follows. Section 2 presents empirical facts motivating our analysis. Section 3 presents the theoretical model, while Section 4 discusses its calibration. Section 5 lays out the baseline FRM economy without indexation. The main results on indexation are in Section 6. Section 7 revisits these indexation results in a model with liquidity-driven mortgage defaults. A series of extensions and robustness checks are presented in Section 8. Section 9 concludes. Model derivations, first order conditions characterizing the solution, details on the computational methods, and additional results are relegated to the appendix.

2 Motivating Empirical Evidence

This section presents motivating empirical evidence for the model that follows. We combine house price data from the Federal Housing Finance Agency with loan performance data from Freddie Mac, to create a quarterly panel at the ZIP-3 level. We present three main facts, illustrated by the plots in Figure 1.

First, house price growth is a key determinant of mortgage defaults and loan losses. Figure 1a displays coefficient estimates from a binned regression of the loan loss rate (the ratio of total loan losses to original principal balance) on each loan's three-digit ZIP-level house

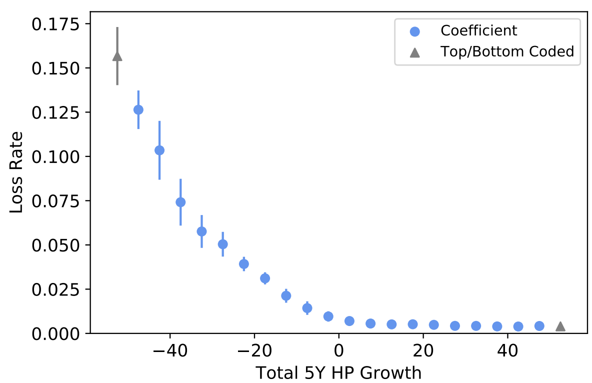
[6] Elenev, Landvoigt, and Van Nieuwerburgh (2016) studies the role the default insurance provided by the government-sponsored enterprises. Gete and Zecchetto (2018) studies the redistributive role of the Federal Housing Agency. Greenwald (2018) studies the interaction between payment-to-income and loan-to-value constraints in a model of monetary shock transmission through the mortgage market, but without default. Favilukis, Ludvigson, and Van Nieuwerburgh (2017) study the role of relaxed down payment constraints in explaining the house price boom. Corbae and Quintin (2014) investigate the effect of risky mortgage innovation in a general equilibrium model with default. Guren and McQuade (2017) study the interaction of foreclosures and house prices in a model with search.

[7] See, e.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Kiyotaki and Moore (1997), and Gertler and Karadi (2011). A second generation of models has added nonlinear dynamics and a richer financial sector. E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), He and Krishnamurty (2013), He and Krishnamurthy (2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016), and Elenev, Landvoigt, and Van Nieuwerburgh (2017).

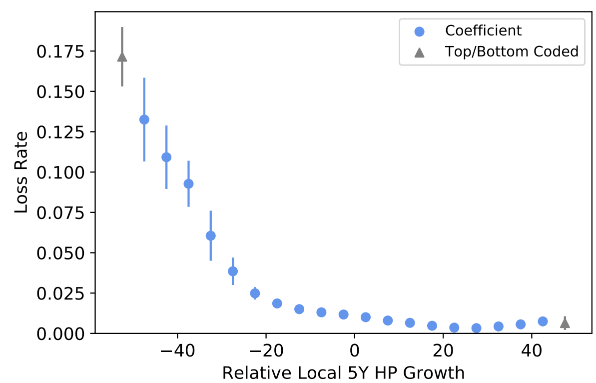
[8] See e.g., Mian and Sufi (2009); Mian, Rao, and Sufi (2013), Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), Fuster and Willen (2015).

price growth over the five years following origination. The estimates show that loan losses are near zero in areas that experience positive house price growth, but are steeply increasing as house prices decline, reaching 15% for areas experiencing house price declines of 50% or more. This link is important to establish in light of findings by Ganong and Noel (2019b) and others demonstrating that liquidity shocks are a key determinant of mortgage default. Our model is able to reconcile these findings because, while negative liquidity events may be necessary for default, they are not sufficient. Since borrowers with positive home equity can choose to sell the property rather than enter foreclosure, while underwater borrowers cannot, we still observe a strong link between household leverage and default. We return to this discussion in Section 7.

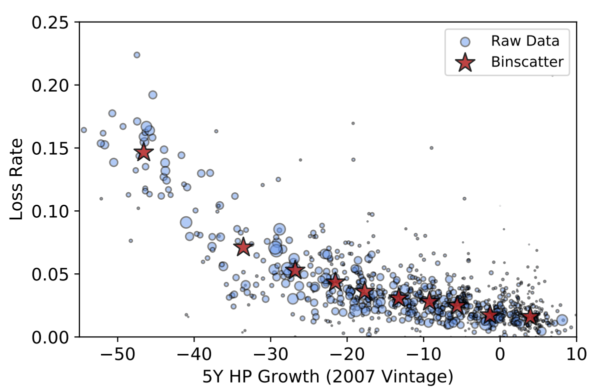
Figure 1: Loan Losses vs. House Prices: Freddie Mac Loan Performance Data



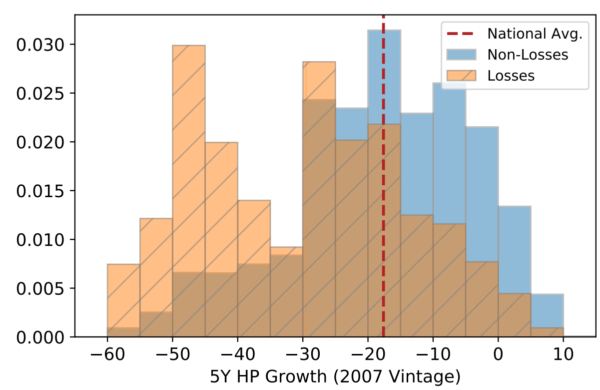
(a) Loss Rate by Total HP Growth



(b) Loss Rate by Local HP Growth



(c) Loss Rate by HP Growth: 2007 Vintage



(d) Losses vs. Repaid Balances

Notes: Source is Freddie Mac Single Family Loan-Level Dataset. See Appendix D for details.

Second, house price growth at the local level is also a key determinant of delinquency and lender losses. This is not obvious, since the total house price growth used in Figure

1a may be correlated with national economic conditions. For example, many of the largest losses occurred during the housing bust period, in the wake of a major financial crisis and recession. To control for this, Figure 1b shows a similar binned regression using the *relative* house price growth after removing the national average, and controlling for time effects, to absorb the influence of the national environment on losses. The resulting estimates using local variation are nearly identical to our original estimates, providing evidence that it is indeed house prices, not confounding national conditions, that drive our findings.

Third, we verify that local house price growth explains much of the variation in outcomes during the recent housing bust, an episode of particular significance to advocates of mortgage indexation. To show this, we restrict our sample to the 2007 vintage of loans — the worst performing vintage in our data. By construction, these loans experienced close to identical national conditions. Figure 1c shows that the link between local house price variation and losses is similar during the crisis period, increasing with the size of the loss, and approaching 15% for the worst-performing areas. Last, Figure 1d compares the house price growth histograms for losses and repaid balances from the 2007 vintage, showing that mortgage loan losses are heavily concentrated in areas that experience average house price declines of 35% or more. Indeed, 77% of losses for this vintage occurred in areas that experienced house price growth below the national average.

Figures D.1 and D.2 in Appendix D show that nearly identical patterns hold for delinquency and foreclosure rates. In all, these results underscore that both national and local house price dynamics are key drivers of mortgage borrower and lender outcomes.

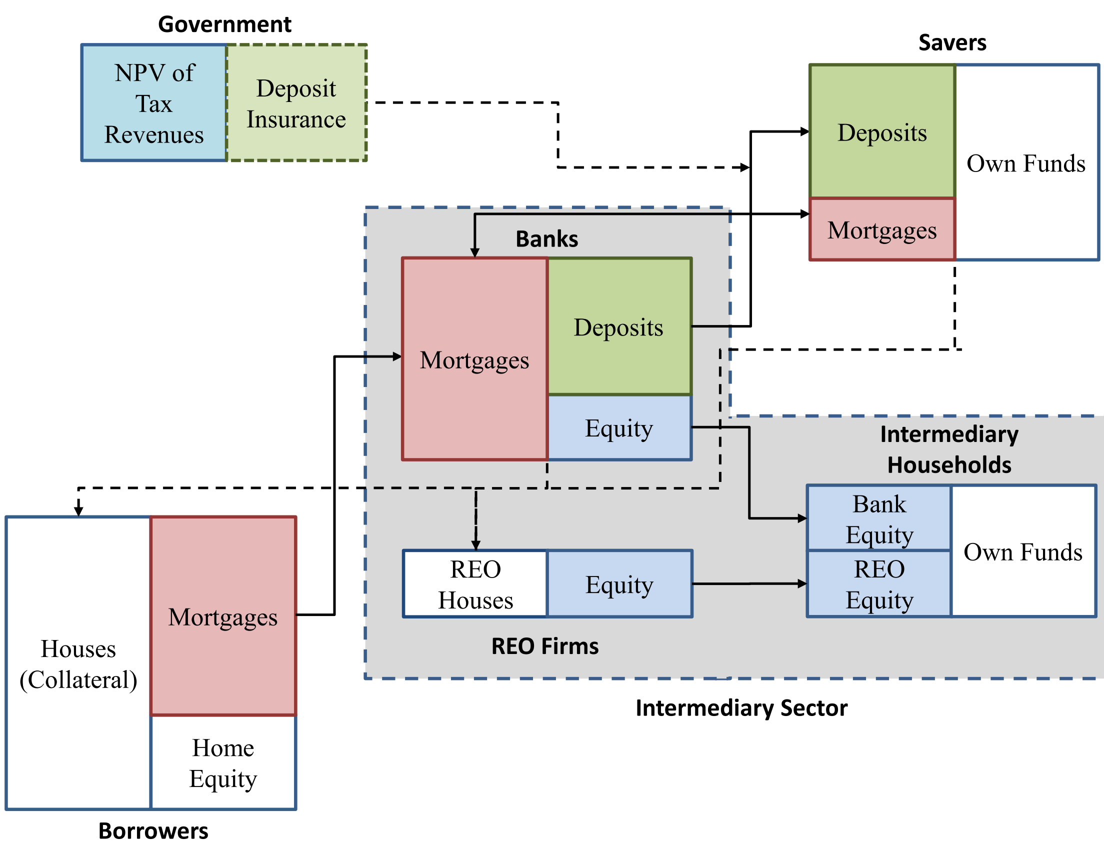
3 Model

3.1 Overview

The model is designed to study how mortgage risk is shared in society. We set up a model of incomplete risk sharing between three types of agents: mortgage borrowers (denoted B), savers (denoted S), and intermediaries (denoted I). Figure 2 graphically represents the model structure.

Savers are relatively patient – hence the saver label – and can invest in both safe assets and risky mortgage debt. Impatient borrowers want to take on long-term mortgage debt. A key friction in our model is that savers have a comparative disadvantage in holding mortgage assets. This creates a role for an intermediation sector with expertise in evaluating mortgage credit and prepayment risk: to transform risky long-term mortgages into safe short-term debt. Intermediaries use their equity capital to buffer mortgage losses. However, the intermediation sector has a limited capacity to absorb losses. It relies on the government as ultimate guarantor of the short-term debt it issues. Thus, mortgage intermediation between borrowers and savers is subject to frictions stemming from the default options of both borrowers and banks. Mortgage default results in foreclosure which come with a resource loss to society. Similarly, bank default causes costly liquidations and the loss of resources.

Figure 2: Model Structure



With traditional fixed-rate mortgages, borrowers bear the majority of house price risk. Large drops in aggregate house prices cause a rise in mortgage foreclosures and loss rates, consistent with the empirical evidence from Section 2. The indexation contracts we study implement a different allocation of risk between borrowers, intermediaries, savers. Indexation of mortgage debt to house prices explicitly shifts house price risk to banks, reducing borrower foreclosures, while potentially making banks more fragile. We study how the welfare of each agent as well as overall welfare are affected by indexation.

The economy is hit with two persistent sources of aggregate risk, described in detail in the calibration section. The first exogenous state fluctuates between a "normal" state and a "crisis" state. The crisis state is associated with a higher mortgage default rate (engineered through the variable $\sigma_{\omega,t}$ defined below) and lower aggregate house prices (engineered through the variable ξ_t below). The second exogenous state is aggregate labor income which fluctuates with the business cycle ($\varepsilon_{y,t}$). We define a "financial recession" as a transition from the normal state to the crisis state combined with a low realization of the aggregate income shock. House prices, safe interest rates, mortgage interest rates, and mortgage default and prepayment rates are all determined in equilibrium. Equilibrium objects depend on the exogenous state of the economy, just described, and on the endogenous wealth distribution. The wealth distribution consists of five continuous state variables: borrower wealth, intermediary wealth, saver wealth, mortgage principal outstanding, and promised mortgage interest payments. We denote the state vector by $\mathcal{S}_t$.

We first characterize the equilibrium with fixed-rate mortgages and calibrate the model to U.S. data. The next part of the analysis studies how indexation of mortgage payments to house prices changes the equilibrium. The key question is whether mortgage indexation can improve risk sharing and overall welfare, which in large part is driven by the performance of the economy during financial recessions.

While borrowers face idiosyncratic house quality shocks and banks idiosyncratic profit shocks, all incompleteness in our model stems from imperfect risk sharing *across* the three household types. We assume perfect risk sharing *among* the agents of a given type. This structure allows for a fraction of borrowers and intermediaries to default in equilibrium, yet to obtain aggregation to a representative household within each type. The upshot is that the wealth distribution, which is a state variable, remains manageable. Others, such as Favilukis et al. (2017) and Guren et al. (2018) allow for imperfect risk-sharing among borrowers, but do not have an intermediary sector. When computing equilibria, they approximate the wealth distribution with a similar number of state variables as in our model. Given our question of how mortgage indexation affects financial fragility and welfare, we use our computational degrees of freedom to provide a richer model of the intermediary sector.

In one special case of the model, discussed in Section 8.5, savers are not allowed to invest directly in mortgage loans. They only indirectly participate in the sharing of mortgage credit and prepayment risk by paying for bank bailouts through taxes and through general equilibrium effects on safe interest rates. Banks adjust their capital structure to manage the increased risk they bear. More generally, the welfare effects of indexation naturally depend on the risk absorption capacity of the intermediary sector.

3.2 Setup: Preferences and Endowments

There is a continuum of agents of each type with population shares χ_j; $\chi_B + \chi_S + \chi_I = 1$. To allow for non-trivial risk premia, an agent of type $j \in \{B, S, I\}$ has preferences following Epstein and Zin (1989), so that lifetime utility is given by

$$U_t^j = \left\{ (1-\beta_j) \left(u_t^j\right)^{1-1/\psi} + \beta_j \left(\mathbb{E}_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{1}$$

$$u_t^j = (C_t^j)^{1-\xi_t} (H_t^j)^{\xi_t} \tag{2}$$

where C_t^j is nondurable consumption and H_t^j is housing services. Borrowers, intermediary households, and savers have different degrees of patience β_j, but all households have the same risk aversion γ and intertemporal elasticity ψ. Naturally, borrowers (and intermediaries) are less patient than savers. The preference parameter ξ_t governs the demand for housing services and varies with the exogenous state of the economy, taking on a low value during the crisis state. We denote by Λ^j the intertemporal marginal rate of substitution or stochastic discount factor of agent j, spelled out in Appendix A.1.

All agents are endowed with non-housing and housing goods. The non-housing endowment Y_t equals a stationary stochastic component $\tilde{Y}_t$ and a deterministic component that grows at a constant rate g, $Y_t = e^{gt}\tilde{Y}_t$, where $\mathbb{E}(\tilde{Y}_t) = 1$, and:

$$\log \tilde{Y}_t = \rho_y \log \tilde{Y}_{t-1} + \sigma_y \varepsilon_{y,t}, \qquad \varepsilon_{y,t} \sim N(0,1). \tag{3}$$

The transitory shocks to the aggregate endowment $\varepsilon_{y,t}$ are the second source of aggregate risk. Each agent type j receives a fixed share s_j of the overall endowment Y_t; this can be interpreted as labor income.

Agents are also endowed with housing. The stock of housing is fixed at $\bar{K}$, and produces housing services that grow at the same rate g as the nondurable endowment. Housing requires a maintenance cost of fraction ν^K of the value of the housing stock. This cost is rebated lump-sum to households.[9] To ensure that the borrowers are the marginal pricers of housing, we fix intermediary and saver demand for housing to be $H_t^I = \bar{K}^I$ and $H_t^S = \bar{K}^S$.

[9] In our endowment economy, housing maintenance stands in for residential investment, which strongly comoves with house prices in the data. The assumption that maintenance is a fraction of current house prices creates this correlation in the model. However, the assumption also means that different versions of the model, for example with mortgage indexation, which have different steady-state house price levels, feature different maintenance expenditure levels. Rebating maintenance expenditure avoids the undesirable effect that higher (lower) house price levels cause lower (higher) consumption.

3.3 Mortgages

Mortgage Contracts. Like in the U.S., mortgages are long-term, nominal, defaultable, prepayable contracts. For tractability, mortgages are modeled as perpetuities with outstanding loan balance and interest payments that decline geometrically. One unit of debt yields payments of $1, \delta, \delta^2, \ldots$ until either prepayment or default; the fraction $(1 - \delta)$ captures the scheduled amortization of principal. Mortgage interest payments can be deducted from taxes. New mortgages have a fixed mortgage rate r_t^*, a principal M_t^*, and are subject to a loan-to-value constraint, shown below in (18), that is applied at origination only.

Refinancing. After the default decision has taken place (explained below), non-defaulting borrowers can choose to refinance. Refinancers first prepay the principal balance on the existing loan before they obtain a new mortgage loan. They re-optimize their housing position. Since borrowers in the model tend to borrow up to their credit limit when taking out new loans, as is typical in reality, adjustments in borrower leverage largely occur at times of refinancing. Refinancing has an important effect on financial fragility because borrower leverage is a key determinant of default.

We assume a transaction cost for obtaining a new mortgage that is proportional to the new loan balance, $\kappa_{i,t} M_t^*$, where $\kappa_{i,t}$ is drawn i.i.d. across borrowers and time from a distribution with CDF Γ_κ. Since these costs largely stand in for non-monetary frictions such as inertia, they are rebated to borrowers and do not impose an aggregate resource cost. Following Greenwald (2018), we assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on $\kappa_{i,t}$ and all current values and expectations of aggregate variables, but cannot depend on the borrower's individual loan characteristics.[10] We guess and verify that the optimal plan for the borrower is to refinance whenever $\kappa_{i,t} \leq \bar{\kappa}_t$, where $\bar{\kappa}_t(\mathcal{S}_t)$ is a threshold cost that makes the borrower indifferent between refinancing and not refinancing and that depends on the entire state of the economy $\mathcal{S}_t$. The fraction of non-defaulting borrowers who choose to refinance is therefore:

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t).$$

Once the threshold cost (or refinancing rate) is known, the total transaction cost per unit of

[10] This assumption keeps the problem tractable by removing the distribution of loans as a state variable while maintaining the realistic feature that an endogenous fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

debt is defined by:

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa \, d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa \, d\Gamma_\kappa.$$

As shown in Appendix A.4, borrowers refinance both to lower their mortgage rate (standard rate refi incentive) and to extract home equity (cash-out refi incentive).

House Quality Shocks. Before deciding whether to refinance a loan, borrowers can choose to default on the loan. Upon default, the housing collateral backing the loan is seized by the intermediary. To obtain an aggregated model in which there is fractional default and the default rate responds endogenously to macroeconomic conditions, we introduce stochastic processes $\omega_{i,t}$ for each borrower i that influence the quality of borrowers' houses. We decompose house quality into two components, $\omega_{i,t} = \omega_{i,t}^L \omega_{i,t}^U$, where $\omega_{i,t}^L$ is a local component that shifts prices in an area relative to the national average while $\omega_{i,t}^U$ is an uninsurable component that shifts an individual house price relative to its local area. The key idea is that payments on shared appreciation mortgages can potentially be indexed to local house prices (e.g., at the MSA- or ZIP-code level). For moral hazard reasons, lenders would be reluctant to index payments to the individual house price component, hence the label uninsurable. Both components are drawn i.i.d. from independent log-normal distributions:

$$\log \omega_{i,t}^L \sim N\left(-\frac{1}{2}\alpha\sigma_{\omega,t}^2, \;\; \alpha\sigma_{\omega,t}^2\right), \tag{4}$$

$$\log \omega_{i,t}^U \sim N\left(-\frac{1}{2}(1-\alpha)\sigma_{\omega,t}^2, \;\; (1-\alpha)\sigma_{\omega,t}^2\right), \tag{5}$$

ensuring that each process has mean unity, and that the local and uninsurable components account for α and $1-\alpha$ of the cross-sectional variance of $\omega_{i,t}$, respectively. The cross-sectional dispersion $\sigma_{\omega,t}$ takes on a low value in normal times and a high value in crisis times, fluctuating with the aggregate state of the economy. While the assumption that local and individual house values are drawn i.i.d. is not realistic, we show in Appendix C.2 that our functional form can be micro-founded based on more realistic AR(1) processes.[11]

Mortgage Indexation. In addition to the standard mortgage contracts defined above, we introduce shared appreciation mortgages (SAMs) whose payments are indexed to house prices. We allow SAM contracts to insure households in two ways. First, mortgage payments

[11] The intuition is that due to perfect insurance within the borrower family and the symmetric form of indexation, swapping the identities of two individual borrower agents is irrelevant. Drawing i.i.d. can therefore be thought of as drawing from a more persistent process, and then randomly reshuffling the identities of the individual borrowers.

can be indexed to the aggregate, i.e. national, house price p_t. In this case, the principal balance and the scheduled principal and interest payments on each existing mortgage loan are multiplied by:

$$\zeta_{p,t} = \left(\frac{p_t}{p_{t-1}}\right)^{\iota_p}. \tag{6}$$

The special cases $\iota_p = 0$ and $\iota_p = 1$ correspond to the cases of no indexation and complete insurance against aggregate house price risk.

Second, mortgage contracts can also be indexed against shocks to the local component $\omega^L_{i,t}$ of house values. The principal balance and payment on the loan backed by a house that experiences local house quality growth $\omega^L_{i,t}$ are multiplied by:

$$\zeta_\omega(\omega^L_{i,t}) = \left(\omega^L_{i,t}\right)^{\iota_\omega}. \tag{7}$$

The special cases $\iota_\omega = 0$ and $\iota_\omega = 1$ correspond to no insurance and complete insurance against cross-sectional local house price risk, respectively.

Indexation of mortgage principal and payments to the aggregate and the local component of house prices can be combined by setting $\iota_p = 1$ and $\iota_\omega = 1$. This is the main case of interest, which we refer to as *regional* indexation. Indeed, regional (MSA- or ZIP-level) house prices are the product of national house prices and the local component of house prices that is orthogonal to the national index.

Mortgage Default. As with refinancing, borrowers must commit to a default plan that can depend in an unrestricted way on $\omega^L_{i,t}, \omega^U_{i,t}$, and the aggregate states, but not on a borrower's individual loan conditions. We guess and verify that the optimal plan for the borrower is to default whenever $\omega^U_{i,t} \leq \bar{\omega}^U_t$, where $\bar{\omega}^U_t$ is the threshold value of uninsurable (individual-level) house quality that makes a borrower indifferent between defaulting and not defaulting. The level of the default threshold depends on the aggregate state $\mathcal{S}_t$, the local component $\omega^L_{i,t}$, and also on the level of mortgage payment indexation. Given $\bar{\omega}^U_t$, the fraction of non-defaulting borrowers $Z_{N,t}$ is:

$$Z_{N,t} = \int \left(1 - \Gamma^U_{\omega,t}(\bar{\omega}^U_t)\right) d\Gamma^L_{\omega,t}, \tag{8}$$

where $\Gamma^U_{\omega,t}$ and $\Gamma^L_{\omega,t}$ are the CDFs of $\omega^U_{i,t}$ and $\omega^L_{i,t}$, respectively. The integral is needed because $\bar{\omega}^U_t$ depends on $\omega^L_{i,t}$. The share of housing held by non-defaulting borrower households is:

$$Z_{K,t} = \int \left(\int_{\omega^U_{i,t} > \bar{\omega}^U_t} \omega^U_{i,t} \, d\Gamma^U_{\omega,t} \right) \omega^L_{i,t} \, d\Gamma^L_{\omega,t}. \tag{9}$$

where inner-most integral contains a selection effect –borrowers only keep their housing when their idiosyncratic quality shock was sufficiently good– while the outer integral again accounts for dependence of $\bar{\omega}^U_t$ on local house quality.

The fractions of principal and interest payments retained, i.e., not defaulted on, are defined by $Z_{M,t}$ and $Z_{A,t}$, respectively, and are given by:

$$Z_{M,t} = Z_{A,t} = \int \int \underbrace{\left(1 - \Gamma^U_{\omega,t}\left(\bar{\omega}^U_t\right)\right)}_{\text{remove defaulters}} \underbrace{\left(\omega^L_{i,t}\right)^{\iota_\omega}}_{\text{indexation}} d\Gamma^L_{\omega,t}. \tag{10}$$

The first term in the integral above removes the fraction of debt that is defaulted on, while the second component adjusts for indexation of debt to local prices.[12]

Equations (11)-(13) describe the evolution of the aggregate outstanding mortgage principal balance, interest payments, and housing stock:

$$M^B_{t+1} = \bar{\pi}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} M^*_t + \delta(1 - Z_{R,t}) Z_{M,t} M^B_t \right] \tag{11}$$

$$A^B_{t+1} = \bar{\pi}^{-1} \zeta_{p,t+1} \left[Z_{R,t} Z_{N,t} r^*_t M^*_t + \delta(1 - Z_{R,t}) Z_{A,t} A^B_t \right] \tag{12}$$

$$K^B_{t+1} = Z_{R,t} Z_{N,t} K^*_t + (1 - Z_{R,t}) Z_{K,t} K^B_t \tag{13}$$

Since mortgages are nominal contracts, dividing by the gross inflation rate $\bar{\pi}$ expresses balance and payments in real terms. Aggregate indexation influences the laws of motion by directly scaling principal and interest payments to aggregate house price growth, through the term $\zeta_{p,t+1}$. Under full aggregate indexation ($\iota_p = 1$), a 20% national house price decline reduces mortgage principal balances and interest payments by 20%. Local indexation, whose direct effects wash out in aggregate, instead influences the default decision, i.e., the default threshold $\bar{\omega}^U_t$, and thereby $Z_{N,t}$, $Z_{M,t}$, $Z_{A,t}$, and $Z_{K,t}$.

Under a standard mortgage contract without indexation ($\iota_p = \iota_\omega = 0$), households strategically default when the current loan-to-value ratio is above one.[13] Default happens when

[12]While $Z_{A,t}$ and $Z_{M,t}$ are identical in the baseline indexation case, it is convenient to define them separately since they will diverge under separate indexation of interest and principal in Section 8.1.

[13]Default occurs when the market value of debt, which includes the option value of waiting to default, exceeds the market value of the housing collateral. In Section 7, we consider a model extension where borrower

they suffer a large house price drop, the origin of which could be at the national, local, or individual level. Indexation to national (local) house prices adjusts the mortgage balance and payments so as to stabilize current leverage, thereby reducing default due to national (local) house price drops. It is straightforward to show that for the limiting case when all cross-sectional house price risk is insurable ($\alpha = 1$) and this risk is fully indexed ($\iota_\omega = 1$, $\iota_p = 1$), we obtain $Z_{N,t} = Z_{M,t} = Z_{A,t} = Z_{K,t} = 1$. Full indexation prevents all mortgage defaults.

Recovery Rate on Foreclosed Mortgages. As discussed below, mortgages can be held by intermediaries ("banks") and by savers. The housing collateral backing a defaulted mortgage is seized by the holder of that mortgage. After paying maintenance on this so called "real estate owned" housing for one period at the higher depreciation rate $\nu^{REO} > \nu^K$, the mortgage holder sells the REO housing to a specialized intermediary, a REO firm, at a price p_t^{REO} determined in equilibrium. The recovery rate X_t on foreclosed mortgages (per unit of principal outstanding) is:

$$X_t = \frac{(1 - Z_{K,t}) K_t^B (p_t^{REO} - \nu^{REO} p_t)}{M_t^B}. \tag{14}$$

Note that X_t is taken as given by each individual mortgage holder. An individual bank does not internalize the effect of its mortgage debt issuance on the overall recovery rate.

PO and IO Strips. After being originated by banks, mortgages can be traded on secondary markets by banks ($j = I$) and savers ($j = S$). Although each mortgage vintage has a different fixed interest rate r_t^* and hence a different secondary market price, we show in Appendix A.2 that any portfolio of loans (vintages) can be replicated using two instruments: an interest-only (IO) strip and a principal-only (PO) strip. Let q_t^A and q_t^M be the market prices of IO and PO strips, respectively. The cash flow CF^j from a generic portfolio of M_t^j units of POs and A_t^j units of IOs equals:

$$CF_t^j = X_t M_t^j + Z_{M,t} \Big(1 - \delta + \delta Z_{R,t}\Big) M_t^j + Z_{A,t} A_t^j, \tag{15}$$

for $j = I, S$. The first term reflects principal recovery from defaulted mortgages. On the non-defaulted mortgage principal, $Z_{M,t} M_t^j$, the investors receives scheduled principal amortization $(1 - \delta)$ as well as unscheduled principal prepayments of the outstanding mortgage

default is driven by liquidity shocks and there is a penalty for strategically defaulting.

balance $\delta Z_{R,t}$. On the IOs, the investor receives $Z_{A,t}A_t^j$ since Z_A is the non-defaulted fraction and each unit of IOs pays 1 in interest in the current period (recall from (12) that the interest rate is folded into the definition of A). The ex-dividend value, EDV, of this portfolio of POs and IOs, i.e., after current-period cash flows have been made, is:

$$EDV_t^j = \delta(1 - Z_{R,t}) \left(q_t^A Z_{A,t} A_t^j + q_t^M Z_{M,t} M_t^j \right). \tag{16}$$

A fraction $\delta(1 - Z_{R,t})Z_{A,t}$ of IOs remain outstanding after default and refinancing decisions and after principal amortization. Each unit is worth q_t^A. Similarly, a fraction $\delta(1 - Z_{R,t})Z_{M,t}$ of POs remain outstanding; each unit is worth q_t^M.

3.4 Borrowers

Given this setup, individual borrowers' problems aggregate to that of a representative borrower. The endogenous state variables to the borrower are the promised payment A_t^B on the stock of all mortgage debt, the outstanding principal balance on all mortgage debt M_t^B, and the stock of borrower-owned housing K_t^B. The representative borrower's choice variables are nondurable consumption C_t^B, housing services consumption H_t^B, the amount of housing K_t^* and new loans M_t^* taken on by refinancing borrowers, the refinancing fraction $Z_{R,t}$, and the default policy $\bar{\omega}_t^U$, which implicitly determines $(Z_{N,t}, Z_{M,t}, Z_{A,t}, Z_{K,t})$.

The borrower maximizes expected utility in (1) subject to the budget constraint:

$$\begin{aligned} C_t^B = & \underbrace{(1-\tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t}\left(Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B\right)}_{\text{net new borrowing}} - \underbrace{(1-\delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1-\tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\ & - \underbrace{p_t Z_{R,t}\left[Z_{N,t}K_t^* - Z_{K,t}K_t^B\right]}_{\text{owned housing}} - \underbrace{\nu^K p_t Z_K K_t^B}_{\text{maintenance}} - \underbrace{\rho_t\left(H_t^B - K_t^B\right)}_{\text{rental housing}} \\ & - \underbrace{\left(\Psi(Z_{R,t}) - \bar{\Psi}_t\right)Z_{N,t}M_t^*}_{\text{net refinancing costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}} + \underbrace{R_t^B}_{\text{maintenance rebate}}, \end{aligned} \tag{17}$$

the loan-to-value constraint:

$$M_t^* \leq \phi^K p_t K_t^*, \tag{18}$$

and the laws of motion (11)-(13). Borrower consumption equals after tax labor income, where τ is the income tax rate, plus net new mortgage borrowing (mortgage principal on new loans minus outstanding principal balance on refinanced loans), minus scheduled principal amortization on outstanding mortgages, minus interest payment on mortgages taking into account the tax shield, minus net new housing purchased by refinancing borrowers, minus housing maintenance expenses to offset depreciation, minus rental expenses (the rental rate ρ_t times the difference between housing services consumed and owned), minus net refinancing costs associated (zero in equilibrium), minus taxes raised on borrowers to pay for intermediary bailouts (defined below in 36), plus a rebate of maintenance costs ($R_t^B = \nu^K p_t Z_K K_t^B$ in equilibrium). Equation (18) caps new mortgage debt at a maximum LTV ratio of ϕ^K. We discuss the borrower's optimality conditions in Appendix A.4.

3.5 Intermediaries

The intermediation sector consists of intermediary households ("bank owners"), mortgage lenders ("banks" for short), and REO firms. The intermediary households are equity holders of both the banks and the REO firms.

Bank Owners. Each period, bank owners receive labor income Y_t^I, and the dividends D_t^I and D_t^{REO} from all banks and REO firms, defined in equations (28) and (30) below. Bank owners choose consumption C_t^I to maximize (1) subject to the budget constraint:

$$C_t^I \leq (1-\tau)Y_t^I + D_t^I + D_t^{REO} - T_t^I - \nu^K p_t H_t^I + R_t^I, \tag{19}$$

where T_t^I are taxes raised on intermediary households to pay for bank bailouts, defined in (36) below, and R_t^I is the lump-sum rebate of maintenance expenditure. Bank owners consume their fixed endowment of housing services each period, $H_t^I = \bar{K}^I$.

Banks' Portfolio Choice. There is a continuum of competitive banks. Banks maximize shareholder value, i.e. the present discounted value of dividends valued at the SDF of their shareholders Λ^I, by optimally choosing new mortgage originations, short-term deposits, and IO and PO positions in the secondary market for mortgage debt.

Let A_t^I and M_t^I denote the bank's holdings of IO and PO strips, respectively, at the start of the period. After all shocks are realized and borrowers have made default decisions, banks originate new mortgages L_t^* to refinancers at interest rate r_t^*. They then re-optimize their portfolio of mortgages on the secondary market. That is, banks *supply* PO and IO strips:

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{M,t}M_t^I \tag{20}$$

$$\hat{A}_t^I = r_t^* L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I. \tag{21}$$

and *demand* a new portfolio of PO and IO strips $\tilde{M}_t^I$ and $\tilde{A}_t^I$, respectively. The market value of the bank's portfolio after portfolio rebalancing is:

$$J_t^I = \underbrace{q_t^A \tilde{A}_t^I}_{\text{IO strips}} + \underbrace{q_t^M \tilde{M}_t^I}_{\text{PO strips}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}} . \tag{22}$$

Next period's beginning-of-period IO and PO strip holdings adjust current end-of-period holdings for inflation and indexation:

$$M_{t+1}^I = \bar{\pi}^{-1} \zeta_{p,t+1} \tilde{M}_t^I, \tag{23}$$

$$A_{t+1}^I = \bar{\pi}^{-1} \zeta_{p,t+1} \tilde{A}_t^I. \tag{24}$$

Banks' net worth at the beginning of period $t+1$ equals the cash flows on its IO and PO portfolio (A_{t+1}^I, M_{t+1}^I) plus the value of that portfolio minus deposit redemptions:

$$W_{t+1}^I = CF_{t+1}^I + EDV_{t+1}^I - \bar{\pi}^{-1} B_{t+1}^I, \tag{25}$$

using equations (15), (16), (23), and (24).

Banks' Problem. At the beginning of each period, aggregate shocks are realized and each bank receives an idiosyncratic profit/loss shock $\epsilon_t^I \sim F_\epsilon^I$, with $\mathrm{E}(\epsilon_t^I) = 0$. A high draw for ϵ_t^I represents a large idiosyncratic loss. The idiosyncratic profit shock captures unmodeled heterogeneity in banks' balance sheets. Then, banks make their optimal default decision. The government seizes the defaulted banks, wipes out the equity holders and makes whole the depositors. Bank owners then start new banks to replace the liquidated banks. Finally, all banks make optimal portfolio choice decisions.

We show in Appendix A.3 that surviving and newly started banks face an identical portfolio choice problem. This property allows for aggregation across all banks. The problem solved by the representative bank, after default decisions have been made, is:

$$V^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right], \tag{26}$$

given the definitions of J_t^I and W_t^I in (22) and (25), and the laws of motion (23) and (24). $F_{\epsilon,t+1}^I \equiv F_\epsilon^I(V^I(W_{t+1}^I, \mathcal{S}_{t+1}))$ is the probability of continuation, and the expectation of the loss realization ϵ_{t+1}^I conditional on continuation is $\epsilon_{t+1}^{I,-} = \mathrm{E}\left[\epsilon_{t+1}^I \,|\, \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1})\right]$. By the law of large numbers, the fraction of defaulting banks in the current period is $1 - F_{\epsilon,t}^I$.

The bank's portfolio choice is subject to a leverage constraint that limits the amount of deposit finance to a fraction ϕ^I of assets:

$$B^I_{t+1} \leq \phi^I \left(q^A_t \tilde{A}^I_t + q^M_t \tilde{M}^I_t \right) \tag{27}$$

Since banks enjoy limited liability and issue insured deposits, they have incentives to take on excessive risk in the form of high leverage. To curb this incentive, the Basel-style regulatory equity capital requirement limits bank leverage to $0 < \phi^I < 1$.

The aggregate dividend paid by banks to their shareholders is:

$$D^I_t = F^I_{\epsilon,t} \left(W^I_t - J^I_t - \epsilon^{I,-}_t \right) - \left(1 - F^I_{\epsilon,t} \right) J^I_t. \tag{28}$$

The first term reflects dividends paid out from non-defaulting banks. Bank shareholders bear the burden of replacing liquidated banks by an equal measure of new banks and seeding them with new capital equal to that of continuing banks (J^I_t); the second term.

Government Bailout. The government bails out defaulted banks at a cost:

$$\text{bailout}_t = \left(1 - F^I_{\epsilon,t} \right) \left[\epsilon^{I,+}_t - W^I_t + \eta EDV^I_t \right], \tag{29}$$

where $\epsilon^{I,+}_t = \mathrm{E}\left[\epsilon^I_t \,|\, \epsilon^I_t > V^I(W^I_t, \mathcal{S}^I_t) \right]$ is the expectation of the idiosyncratic loss ϵ^I_t conditional on default. The government absorbs the negative net worth of the defaulting banks, $\epsilon^{I,+}_t - W^I_t$. The last term captures deadweight losses from bank bankruptcies, which are a fraction η of the mortgage assets seized from the bankrupt banks. The government bailout always makes depositors whole. This deposit insurance is what makes deposits risk-free.

REO Firm's Problem. There is a continuum of competitive REO firms that are owned and operated by intermediary households. REO firms maximize the present discounted value of dividends by choosing how many foreclosed properties to buy, I^{REO}_t:

$$D^{REO}_t = \underbrace{\left[\rho_t + (S^{REO} - \nu^{REO}) p_t \right] K^{REO}_t}_{\text{REO net income}} - \underbrace{p^{REO}_t I^{REO}_t}_{\text{REO investment}}. \tag{30}$$

REO firms earn revenue from renting foreclosed homes to borrowers and gradually selling them back to borrowers at an exogenous rate S^{REO}. REO firms must pay for maintenance $\nu^{REO} p_t K^{REO}_t$, which unlike regular housing maintenance is not rebated and thus constitutes an aggregate resource cost. This cost is the deadweight loss from mortgage foreclosures. The law of motion of the REO housing stock is:

$$K_{t+1}^{REO} = (1 - S^{REO})K_t^{REO} + I_t^{REO}. \tag{31}$$

3.6 Saver's Problem

Savers can invest in risk-free debt and risky mortgage debt. To capture the comparative disadvantage of savers in holding mortgages relative to banks, savers face a cost of holding mortgages. When mortgages default, savers sell the collateral backing the defaulted mortgages to REO firms after one period of maintenance, just like banks do.

Savers enter the period with net worth W_t^S. They sell their holdings of mortgages into the secondary market; call this supply $(\hat{A}_t^S, \hat{M}_t^S)$. They then form an optimal portfolio of safe assets and mortgages $(\tilde{A}_t^S, \tilde{M}_t^S)$. For simplicity, we assume that savers only buy and sell mortgages in fixed combinations of IO and PO strips. Denote the post-trade value of their portfolio by:

$$J_t^S = q_t^A \tilde{A}_t^S + q_t^M \tilde{M}_t^S + q_t^f B_{t+1}^S. \tag{32}$$

The laws of motion (23) and (24) equally apply to saver holdings. Net worth at the beginning of period $t+1$ equals the cash flows on its IO and PO portfolio (A_{t+1}^S, M_{t+1}^S) plus the ex-dividend value of that portfolio minus deposit redemptions:

$$W_{t+1}^S = CF_{t+1}^S + EDV_{t+1}^S + \bar{\pi}^{-1} B_{t+1}^S, \tag{33}$$

using equations (23), (24), (15), and (16).

The savers' problem can also be aggregated, so that the representative saver chooses nondurable consumption C_t^S, holdings of safe assets B_t^S, and mortgages $\tilde{M}_t^S$, to maximize expected lifetime utility (1) subject to the budget constraint:

$$C_t^S \leq (1-\tau)Y_t^S + W_t^S - J_t^S - \frac{\varphi_0}{\varphi_1}\left(\tilde{M}_t^S\right)^{\varphi_1} - T_t^S - \nu^K p_t H_t^S + R_t^S. \tag{34}$$

and restrictions that safe debt and mortgage holdings must be positive: $B_t^S \geq 0$ and $\tilde{M}_t^S \geq 0$. Savers consume their fixed endowment of housing services each period, $H_t^S = \bar{K}^S$, on which they pay maintenance expenses that are rebated lump-sum. Savers incur a cost for holding mortgages ($\varphi_0 > 0$) which is increasing in the amount of mortgage debt they own ($\varphi_1 > 0$); this cost is rebated lump-sum as part of R_t^S so that it does not represent a resource loss to society. This holding cost represents the comparative disadvantage of savers relative to banks for holding (screening and monitoring) mortgage debt.

3.7 Government

Discretionary government spending equals income taxes net of the mortgage interest deduction:

$$G_t = \tau(Y_t - Z_{A,t} A_t^B).$$

To finance bank bailout expenses in (29), the government issues risk-free short-term debt that trades at the same price as deposits. To service this debt, the government levies lump-sum taxes T_t^j on households of type j in period t. Total tax revenue from lump-sum taxation is $T_t = T_t^B + T_t^I + T_t^S$. Therefore, if B_t^G is the amount of government bonds outstanding at the beginning of t, the government budget constraint satisfies:

$$\bar{\pi}^{-1} B_t^G + \text{bailout}_t = q_t^f B_{t+1}^G + T_t. \tag{35}$$

Lump-sum taxes are levied in proportion to population shares χ_j and at a rate τ_L:

$$T_t^j = \chi_j \tau_L \left(\bar{\pi}^{-1} B_t^G + \text{bailout}_t \right), \ \forall j \in \{B, I, S\}. \tag{36}$$

When $\tau_L < 1$, this formulation implies gradual repayment of government debt following a bailout. When $\tau_L = 1$, the bailout is financed entirely with current taxes.[14]

3.8 Equilibrium

Given a sequence of endowment and crisis shock realizations $[\varepsilon_{y,t}, (\sigma_{\omega,t}, \xi_t)]$, a competitive equilibrium is a sequence of saver allocations $(C_t^S, B_{t+1}^S, \tilde{M}_t^S, \tilde{A}_t^S, \hat{M}_t^S, \hat{A}_t^S)$, borrower allocations $(C_t^B, H_t^B, M_t^B, A_t^B, K_t^B, K_t^*, M_t^*, Z_{R,t}, \bar{\omega}_t^U)$, intermediary allocations $(C_t^I, M_t^I, A_t^I, K_t^{REO}, W_t^I, L_t^*, I_t^{REO}, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I)$, and prices $(r_t^*, q_t^M, q_t^A, q_t^f, p_t, p_t^{REO}, \rho_t)$, such that borrowers, intermediaries, and savers optimize, and markets clear:

$$\begin{aligned}
\text{New mortgages:} \quad & Z_{R,t} Z_{N,t} M_t^* = L_t^* \\
\text{PO strips:} \quad & \tilde{M}_t^I + \tilde{M}_t^S = \hat{M}_t^I + \hat{M}_t^S \\
\text{IO strips:} \quad & \tilde{A}_t^I + \tilde{A}_t^S = \hat{A}_t^I + \hat{A}_t^S \\
\text{Deposits and Gov. Debt:} \quad & B_{t+1}^I + B_{t+1}^G = B_{t+1}^S \\
\text{Housing Purchases:} \quad & Z_{R,t} Z_{N,t} K_t^* = S^{REO} K_t^{REO} + Z_{R,t} Z_{K,t} K_t^B
\end{aligned}$$

[14] Equations (35) and (36) combined imply that new bonds issued in t are $B_{t+1}^G = (1 - \tau_L)(q_t^f)^{-1} \left(\bar{\pi}^{-1} B_t^G + \text{bailout}_t \right)$. The case $\tau_L = 1$ implies $B_t^G = 0, \ \forall t$. To ensure stationarity of the government debt balance, τ_L needs to be large enough relative to the average risk-free rate. We verify that this is the case in our quantitative exercises. Results for $\tau_L < 1$ are discussed in Section 8.6.

$$
\begin{aligned}
\text{REO Purchases:} \quad & I_t^{REO} = (1 - Z_{K,t})K_t^B \\
\text{Housing Services:} \quad & H_t^B = K_t^B + K_t^{REO} = \bar{K}^B \\
\text{Resources:} \quad & Y_t = C_t^B + C_t^I + C_t^S + G_t + DWL_t^b + MAINT_t \\
DWL_t^b = \quad & \left(1 - F_{\epsilon,t}^I\right) \eta\delta(1 - Z_{R,t}) \left(Z_{A,t} q_t^A A_t^I + Z_{M,t} q_t^M M_t^I\right) \\
MAINT_t = \quad & \nu^{REO} p_t \left[K_t^{REO} + (1 - Z_{K,t})K_t^B\right]
\end{aligned}
$$

The resource constraint states that the endowment Y_t is spent on nondurable consumption, government consumption, deadweight losses from bank failures, and housing maintenance. Maintenance consists of payments for houses owned by REO firms, K_t^{REO}, or newly bought by REO firms from foreclosed borrowers $(1 - Z_{K,t})K_t^B$; recall that regular maintenance by households is rebated and thus does not affect the resource constraint.

Appendix B contains a description of the system of equations that characterizes equilibrium and the numerical solution method. The model is solved using global projection methods. Since the integrals (9) and (10) lack a closed form, we evaluate them using Gauss-Hermite quadrature with 11 nodes in each dimension.

4 Calibration

This section describes the calibration procedure for key variables, summarizing the full set of parameter values in Table 1. The model is calibrated at quarterly frequency. All data are for the period 1991.Q1-2016.Q1, the longest period of mortgage foreclosure data. Data sources are detailed in Appendix D.2.

Exogenous Shock Processes. Aggregate endowment shocks in (3) have quarterly persistence $\rho_y = .977$ and innovation volatility $\sigma_y = 0.81\%$. These are the observed persistence and innovation volatility of log real per capita labor income. This AR process is discretized as a five-state Markov Chain, following the Rouwenhorst (1995) method. We abstract from long-run endowment growth $(g = 0)$. The average level of aggregate income (GDP) is normalized to 1. The income tax rate is $\tau = 0.147$, the observed ratio of personal income tax revenue to personal income.

The crisis state follows a two-state Markov Chain, with state 0 indicating normal times, and state 1 indicating crisis. The probability of staying in the normal state in the next quarter is $\Pi_{00} = 97.5\%$ and the probability of staying in the crisis state in the next quarter is $\Pi_{11} = 92.5\%$. Under these parameters, the economy spends 3/4 of the time in the normal state and 1/4 in the crisis state. This matches the fraction of time between 1991.Q1 and 2016.Q4 that the U.S. economy was in the foreclosure crisis, and implies an average duration of the normal state of 10 years, and an average duration of the crisis state of 3.33 years.[15] These transition probabilities are independent of the aggregate endowment state. The normal state has $\bar{\sigma}_{\omega,0} = 0.200$ and crisis state has $\bar{\sigma}_{\omega,1} = 0.250$. These numbers allow the model to match an average mortgage default rate of 0.5% per quarter in expansions and of 2.15% per quarter in financial recessions, which are periods defined by low endowment growth and high uncertainty. The unconditional mortgage default rate in the model is 0.97%. In the data, the average mortgage delinquency rate is 1.05% per quarter: 0.7% in normal times and 2.3% during the foreclosure crisis.

Local House Price Process. We calibrate the cross-sectional dispersion of the local housing quality process using MSA-level house prices indices from FHFA. Specifically, we run the annual panel regression:

$$\log HPI_{i,t} = \phi_t + \psi_i + \rho_\omega^{ann} \log HPI_{i,t-1} + \varepsilon_{i,t} \tag{37}$$

where i indexes the MSA, and t indexes the year, and ϕ_t and ψ_i are year and MSA fixed effects.[16] The quarterly persistence is computed as $\rho_\omega = (\rho_\omega^{ann})^{1/4}$, which we estimate to be 0.977. Since this persistence parameter only matters for the indexation of local house price risk, it is appropriate to calibrate this parameter only to local house price data. To calibrate α, the share of house price variance at the local/regional level, we use (37) to compute the implied unconditional variance $\text{Var}(\omega_{i,t}^L) = \text{Var}(\varepsilon_{i,t})/(1 - (\rho_\omega^{ann})^2)$, which delivers an unconditional standard deviation at the MSA level of 11.5%. We set $\alpha = 0.25$, which generates an unconditional volatility of local house prices of 10.6% close to the data. Given our calibration for $\sigma_{\omega,t}$, it implies that the standard deviation of house prices is 10% in the model in normal times and 12.5% in financial recessions.

Demographics, Income, and Housing Shares. We split the population into mortgage borrowers, savers, and bank owners as follows. We use the 1998 Survey of Consumer Finances to define for every household a loan-to-value ratio. This ratio is zero for renters and for households who own their house free and clear. We define mortgage borrowers to be those

[15] Using a longer time series for the U.S. (1870-2011), Jordà, Schularick, and Taylor (2017) find that the U.S. was in a financial crisis in 20% of the years. For a larger sample of 17 developed nations, they find that one quarter of recessions are financial crises. The same is true in our model. A financial recession, which is the combination of a decline in aggregate labor income and a crisis occurs in 7.5% of our model periods.

[16] Using quarterly house price data instead results in very similar estimates of the cross-sectional dispersion.

Table 1: Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Target/Source
		Technology	
Agg. income persistence	ρ_{TFP}	0.977	Real per capita labor income BEA
Agg. income st. dev.	σ_{TFP}	0.008	Real per capita labor income BEA
Profit shock st. dev.	σ_{ϵ}	0.065	FDIC bank failure rate
Transition: Normal $\rightarrow$ Normal	Π_{00}	0.975	Avg. length = 10Y
Transition: Crisis $\rightarrow$ Crisis	Π_{11}	0.925	25% of time in crisis state
		Demographics and Income	
Fraction of borrowers	χ_B	0.343	SCF 1998 population share LTV$>$.30
Fraction of intermediaries	χ_I	0.050	Stock holders in SCF 1998
Borr. inc. and housing share	s_B	0.470	SCF 1998 income share LTV$>$.30
Intermediary inc. and housing share	s_I	0.062	Income stock holders in SCF 1998
		Housing and Mortgages	
Housing stock	$\bar{K}$	1	Normalization
Housing XS persistence	ρ_{ω}	0.977	FHFA MSA-level regression
Housing XS dispersion (Normal)	$\bar{\sigma}_{\omega,0}$	0.200	Mortg. delinq. rate U.S. banks, no crisis
Housing XS dispersion (Crisis)	$\bar{\sigma}_{\omega,1}$	0.250	Mortg. delinq. rate U.S. banks, crisis
Local share of XS dispersion	α	0.25	FHFA MSA-level regression
Inflation rate	$\bar{\pi}$	1.006	2.29% CPI inflation
Mortgage duration	δ	0.996	Principal amortization on 30-yr FRM
Prepayment cost mean	μ_{κ}	0.370	Greenwald (2018)
Prepayment cost scale	s_{κ}	0.152	Greenwald (2018)
LTV limit	ϕ^K	0.850	LTV at origination
Maint. cost (owner)	ν^K	0.616%	BEA Fixed Asset Tables
		Intermediaries	
Bank regulatory capital limit	ϕ^I	0.930	Financial sector leverage limit
Deadweight cost of bank failures	η	0.050	Bank receivership expense rate
Maint. cost (REO)	ν^{REO}	0.022	REO discount: $p_{ss}^{REO} / p_{ss} = 0.725$
REO sale rate	S^{REO}	0.167	Length of foreclosure crisis
		Savers	
Mortgage holding cost, coeff.	φ_0	0.200	Avg. HH sector's share m. debt, FoF
Mortgage holding cost, expon.	φ_1	5.000	Vol. of HH sector's share m. debt, FoF
		Preferences	
Borr. discount factor	β_B	0.950	Borrower LTV, SCF
Intermediary discount factor	β_I	0.950	Equal to β_B
Depositor discount factor	β_D	0.998	3% nominal short rate (annual)
Risk aversion	γ	2.000	Standard value
EIS	ψ	1.000	Standard value
Housing preference (Normal)	$\bar{\xi}_0$	0.210	Borrower hous. expend./income
Housing preference (Crisis)	$\bar{\xi}_1$	0.160	HP growth volatility
		Government	
Income tax rate	τ	0.147	Personal tax rate BEA
Bailout taxation rate	τ_L	1.0	Tractability, relaxed in Section 8.6

households with a LTV ratio of at least 30%.[17] Those households make up 34.3% of households ($\chi_B = .343$) and earn 46.9% of labor income ($s_B = .469$). For parsimony, we set all housing shares equal to the corresponding income share. Since the aggregate housing stock $\bar{K}$ is normalized to 1, $\bar{K}^B = .469$.

To split the remaining households into savers and intermediary households, we again turn to the 1998 SCF and define a household's risky share as the ratio of direct and indirect equity holdings plus net business wealth to financial assets. We define intermediary households, the "shareholders" in the model, as those households with a risky share above a cutoff. We choose the cutoff such that bank owners' population share is 5%, implying a risky share cutoff of 68.2%. The share of labor income for this group in the SCF is equal to $s_I = 6.2\%$. In Section 8.5, we check the sensitivity of our results to the relative size of intermediary households, which influences banks' risk absorption capacity. The savers make up the remaining $\chi_S = 60.7\%$ of the population, and receive the remaining $s_S = 46.9\%$ of labor income and of the housing stock.

Prepayment Costs. For the prepayment cost distribution, we assume a mixture distribution, so that with probability 3/4, the borrower draws an infinite prepayment cost, while with probability 1/4, the borrower draws from a logistic distribution, yielding

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t) = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(\frac{\bar{\kappa}_t - \mu_\kappa}{\sigma_\kappa}\right)}$$

The calibration of the parameters follows Greenwald (2018).[18] The parameter σ_κ determining the sensitivity of prepayment to equity extraction and interest rate incentives, is set to that paper's estimate (0.152), while the parameter μ_κ is set to match the average quarterly prepayment rate of 3.76% found in that exercise.

Mortgages. We set $\delta = .99565$ to match the fraction of principal U.S. households amortize on mortgages.[19] The maximum loan-to-value ratio at mortgage origination is $\phi^B = 0.85$,

[17] Those households account for 88.2% of all mortgage debt and 81.6% of all mortgage payments.

[18] The parameters are fit to minimize the forecast error $LTV_t = Z_{R,t} LTV_t^* + (1 - Z_{R,t})\delta HPA_t^{-1} LTV_{t-1}$, where LTV_t is the ratio of total mortgage debt to housing wealth, LTV_t^* is LTV at origination, and HPA_t is growth in house values. See Greenwald (2018), section 4.2.

[19] The average duration of a 30-year fixed-rate mortgage is about 7 years. This low duration is mostly the result of early prepayments. The parameter δ captures amortization absent refinancing. Put differently, households are paying off a much smaller fraction of their mortgage principal than 1/7th each year in the absence of prepayment. A quarterly value of $\delta = .99565$ implies that 1.73% of principal is paid off in the first year of the mortgage, matching the first-year principal reduction on a 30-year FRM with a 4.25% rate.

consistent with average mortgage underwriting norms.[20] Inflation $\bar{\pi}$ is set to the observed 0.57% per quarter (2.29% per year) over our sample period.

Banks. We set the maximum leverage that banks may take on at $\phi^I = 0.930$, following Elenev et al. (2017), to capture the historical average leverage ratio of the leveraged financial sector. The idiosyncratic profit shock that hits banks has standard deviation of $\sigma_\epsilon = 6.50\%$ per quarter. This delivers a bank failure rate of 0.30% per quarter, consistent with historical bank failure rate data from the FDIC. We assume a deadweight loss from bank bankruptcies equal to $\eta = 5.00\%$ of bank assets. Based on a study of bank failures from 1986 until 2007 (Bennett and Unal, 2015), the FDIC estimates that direct expenses of resolution for failed banks that are liquidated are 4.88% of assets.

Housing Maintenance and REOs. We set the regular housing maintenance cost equal to $\nu^K = 0.616\%$ per quarter or 2.46% per year. This is the average of the ratio of current-cost depreciation of privately-owned residential fixed assets to the current-cost net stock of privately-owned residential fixed assets at the end of the previous year (BEA Fixed Asset Tables 5.1 and 5.4). We calibrate the maintenance cost in the REO state to $\nu^{REO} = 2.20\%$ per quarter. It delivers REO housing prices that are 23.1% below regular housing prices on average. This is close to the observed fire-sale discounts (losses-given-default) reported by Fannie Mae and Freddie Mac during the foreclosure crisis.

We assume that $S^{REO} = 0.167$ so that 1/6th of the REO stock is sold back to the borrower households each quarter. It takes eight quarters for 75% of the REO stock to roll off. This generates REO crises that take some time to resolve, as they did in the data.

Savers. Savers' holding cost of mortgage securities has two parameters, the cost shifter φ_0 and the elasticity parameter φ_1. We set $\varphi_0 = 0.200$ to target an average saver share of mortgage holdings of 15%, and we set $\varphi_1 = 5.000$ to target a volatility of this share of 3%. We arrive at these targets by calculating the fraction of mortgage debt held outside the levered financial sector using the Financial Accounts of the United States, as detailed in Appendix D.2. The model produces an average share of 14.5% with a volatility of 2.55%.

[20] The average LTV of purchase mortgages originated by Fannie and Freddie was in the 80-85% range during our sample period. However, that does not include second mortgages and home equity lines of credit. Our limit is a combined loan-to-value limit (CLTV). It also does not capture the lower down payments on non-conforming loans that became increasingly prevalent after 2000. Keys, Piskorski, Seru, and Vig (2012) document CLTVs on non-conforming loans that rose from 85% to 95% between 2000 and 2007.

Preferences. All agents have the same risk aversion coefficient of $\gamma_j = 2.000$ and intertemporal elasticity of substitution coefficient $\psi = 1$. These are standard values in the literature. We choose the value of the housing preference parameter in normal times $\bar{\bar{\xi}}_0 = 0.210$ to match a ratio of housing expenditure to income for borrowers of 19%, a common estimate in the housing literature.[21] The model produces an expenditure ratio of 19.5%. To induce an additional house price drop, we set $\bar{\bar{\xi}}_1 = 0.16$ in the crisis states. This additional variation yields a volatility of quarterly log national house price growth of 1.64%, matching the 1.66% in the data (Case-Shiller national home price index, deflated by PCE, 1991.Q1 - 2016.Q4).

For the time discount factors, we set $\beta^B = \beta^I = 0.950$ to target average borrower mortgage debt to housing wealth (LTV) of 64.3%, close to the corresponding value 61.6% for the borrower population in the 1998 SCF. We set the discount rate of savers $\beta^D = 0.998$ to exactly match the observed nominal short rate of 3.1% per year or 0.76% per quarter.

With these parameters, the model generates a ratio of housing wealth to quarterly income for borrowers of 8.27, close to the 8.67 ratio for borrowers in the 1998 SCF. The model somewhat overstates total housing wealth, which represents about 212.6% of annual GDP in the model and 153% in the data. This discrepancy is an artifact of giving all agents the same housing to income ratio in the model, while the "borrower" type holds relatively more housing in the data than the other groups. In equilibrium, only borrower holdings of housing are relevant, so the quantitative effect of exaggerating total housing wealth is minimal.

Government. We set the income tax rate τ in the model to match the average effective personal tax rate of 14.7% as reported by the BEA. We further set the fraction of bailout expenses funded through lump-sum taxation in the same period, τ_L, to 100%. This assumption guarantees that the outstanding balance of government debt B_t^G is always zero, which avoids government debt as state variable. In Section 8.6, we test the sensitivity of our quantitative conclusions to a different taxation regime with a positive amount of government debt. We find that the assumption of instantaneous taxation does not significantly affect our quantitative conclusions about the different indexation schemes.

5 Fixed-rate Mortgage Benchmark

To establish a benchmark for the indexation results in the next section, we start by solving a model without indexation (No Index model). Mortgages are of the standard fixed-rate

[21] Piazzesi, Schneider, and Tuzel (2007) obtain estimates between 18 and 20 percent based on national income account data (NIPA) and consumption micro data (CEX). Davis and Ortalo-Magné (2011) obtain a ratio of 18% after netting out 6% for utilities from the median value of 24% across MSAs using data on rents.

variety. Of particular importance is how the model behaves in a financial recession.

Unconditional Moments. We conduct a long simulation of the model and display the resulting averages of key prices and quantities in the first column of Table 2. As discussed in the calibration section, the model generates an unconditional average mortgage debt to annual income ratio, LTV ratio among mortgage borrowers, and mortgage default, loss-given-default, and refinancing rates that all match the data. The maximum LTV constraint, which only applies at origination and caps the LTV at 85% always binds in simulation, consistent with the overwhelming majority of borrowers taking out new loans up to the limit.

On the intermediary side, the model matches the leverage ratio of the levered financial sector, which is 92.98% in the model. Banks' regulatory capital constraints bind in 100.00% of the periods in the baseline model. Bank equity capital represents 4.4% of annual GDP (17.6% of quarterly GDP) and 7.04% of bank assets in the model. Bank deposits (that go towards financing mortgage debt) represent just over 50.1% of annual GDP (200.3%/4). Bank dividends are 0.9% of GDP. The model generates a substantial amount of financial fragility. The bank default rate is 0.30% per quarter or 1.2% per year. Deadweight losses from bank bankruptcies represent 0.03% of GDP in an average year.

REO firms represent the other part of the intermediary sector. They spend 0.31% of GDP on housing maintenance, and pay 0.5% of GDP in dividends to their owners. REO firms earn high returns from investing in foreclosed properties and selling them back to the borrowers: the return on equity is 5.4% per quarter.[22]

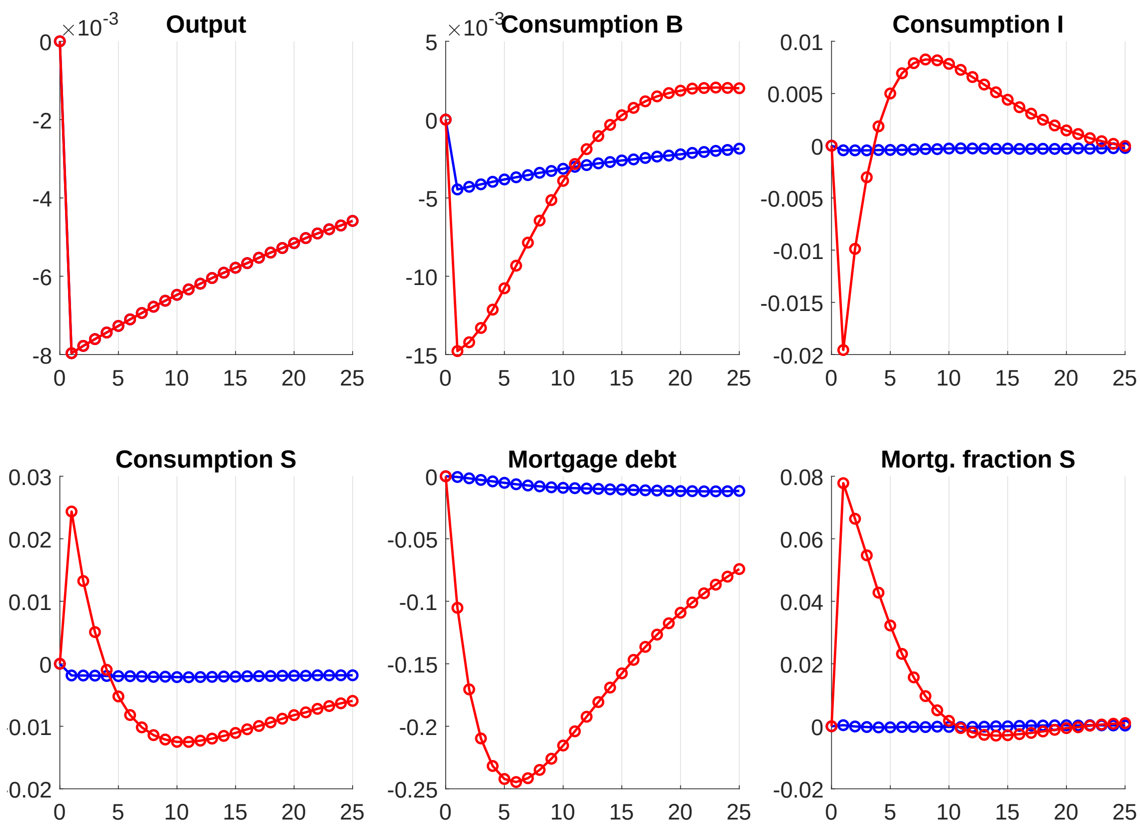
The mortgage rate, which was not directly targeted in the calibration, exceeds the short rate by 80 bps per quarter. This is close to the average spread between the 30-year fixed-rate mortgage rate and the 3-month T-bill rate of 89 bps per quarter for 1991–2016. The mortgage spread compensates for time value of money, expected credit losses, and for interest rate, prepayment, and default risk. The expected excess return (risk premium) earned by banks on mortgages is 40 bps per quarter.

Financial Crises. To understand risk-sharing patterns in the benchmark FRM model, it is instructive to study how the economy behaves in a financial versus a non-financial recession. We define a non-financial recession event as a one standard deviation drop in aggregate income while the economy remains in the normal (non-crisis) state. In a financial recession,

[22]This return on equity in the model mimics the high returns earned by private equity firms. For example, the PE fund Blackstone bought nearly 100,000 single-family homes in foreclosure during the financial crisis and exited that investment through an IPO of Invitation Homes recently. IRR targets of 20% per year are not uncommon for opportunistic real estate private equity funds.

the economy experiences the same fall in aggregate income, but also transitions from the normal into the crisis state, leading to an increase in house value uncertainty ($\bar{\sigma}_{\omega,0} \to \bar{\sigma}_{\omega,1}$) and a decrease in housing utility ($\bar{\xi}_0 \to \bar{\xi}_1$). We simulate many such recessions to average over the endogenous state variables (i.e., the wealth distribution). Figures 3 and 4 plot the impulse-response functions, with financial recessions indicated by red circles and non-financial recessions in blue.[23] By construction, the blue and red lines coincide in the top left panel of Figure 3.

Figure 3: Financial vs. Non-financial Recessions: Benchmark Model (part 1)



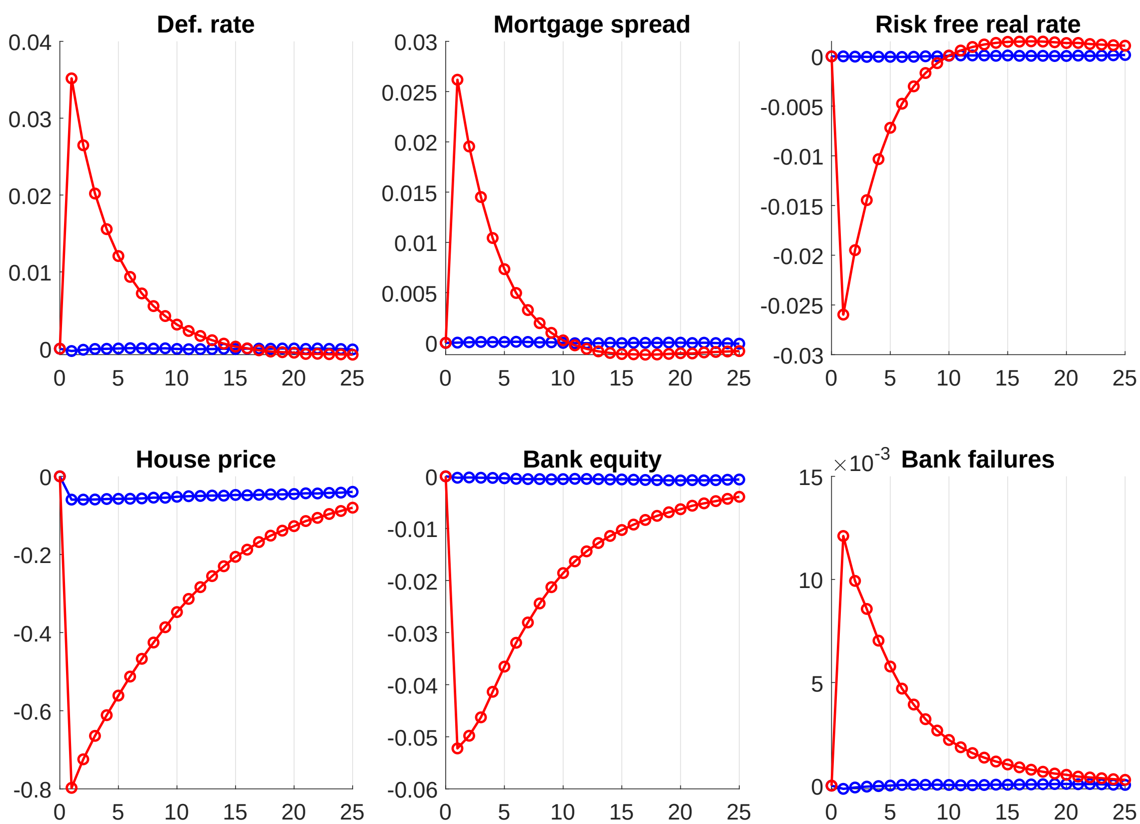
Blue: non-financial recession, **Red:** financial recession. Plots report deviations in levels from the ergodic steady state.

Figure 4 shows that a financial crisis results in a significant increase in mortgage defaults. The risk on existing mortgages goes up, but the fixed interest rates do not, causing the value of bank assets to fall. Faced with reduced equity, some banks fail, while the remaining ones are forced to delever in the wake of the losses they suffer, substantially shrinking both mortgage assets and deposit liabilities. As banks shed mortgage assets, savers expand their

[23]The simulations underlying these generalized IRF plots are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the non-crisis state ($\bar{\sigma}_{\omega,0}$, $\bar{\xi}_0$).

share of outstanding debt by over 50% relative to their pre-crisis position. To induce saver households to reduce deposits demand, the real interest rate falls (Figure 3). Savers' drop in deposit holdings is less than fully offset by their increase in mortgage holdings, and as a result saver consumption rises. Intermediary consumption drops heavily, as the owners of the intermediary sector absorb losses from their banks. Borrower consumption also falls. Faced with higher mortgage rates, borrowers cut back on new borrowing, and must help pay for the bank bailouts through higher taxes. After the shock, the economy slowly recovers as high excess returns on mortgages eventually replenish bank equity.

Figure 4: Financial vs. Non-financial Recessions: Benchmark Model (part 2)



Blue: non-financial recession, **Red:** financial recession. Plots report deviations in levels from the ergodic steady state.

6 Main Results on Mortgage Indexation

Our main exercise introduces indexation of mortgage principal and interest payments to house prices, and compares the resulting equilibrium to that in the No-Index economy.

While the empirically relevant case combines indexation to both aggregate and local house price shocks ($\iota_p = 1$ and $\iota_\omega = 1$) – the Regional model, – it turns out to be conceptually useful to break up this case into an Aggregate model, with only indexation to national house prices ($\iota_p = 1$ and $\iota_\omega = 0$), and a Local model, with only indexation to the component of regional house prices that is orthogonal to the national index ($\iota_p = 0$ and $\iota_\omega = 1$). The two forms of indexation yield sharply different economic implications. Table 2 presents unconditional moments for the Aggregate, Local, and Regional models in columns 2, 3, and 4, respectively.

6.1 Aggregate Indexation

The conjecture in the literature is that indexing mortgage payments to aggregate house prices should reduce mortgage defaults and improve borrower's ability to smooth consumption. Perhaps surprisingly, we find that this conjecture does not hold up in general equilibrium. To the contrary, by adding to financial fragility (bank default rates nearly quadruple), aggregate indexation destabilizes borrower consumption (its volatility increases nearly 240%) while leaving mortgage default rates unchanged.

To understand this, Figure 5 compares financial recessions in the No-Index (black line) and Aggregate (red line) models. Under aggregate indexation, banks find themselves exposed to increased risk through their loan portfolio, whose cash flows now fluctuate directly with aggregate house price movements. Although banks optimally choose to hold slightly more capital, the extra buffer is insufficient to protect their equity from the much greater risks they face. The rise in default risk increases the value of the bankruptcy option. Left with a trade-off between preserving franchise value and exploiting limited liability, banks optimally lean more toward their option to declare bankruptcy and saddle the government with the losses.

The combination of increased risk and the absence of precautionary capital means that the share of bank defaults upon entering a financial recession is vastly larger in the Aggregate economy, with nearly 40% of banks failing. This spike in bank failures necessitates a wave of government bailouts of bank deposits, placing a large tax burden of 0.6% of quarterly GDP on the population. This tax obligation depresses borrower consumption and housing demand, contributing to a larger drop in house prices relative to the benchmark. The breakdown in intermediation and risk sharing is reflected in the upward spike in depositor consumption while at the same time borrower and intermediary households have to sharply cut consumption.[24] Savers' holdings of mortgage debt provide some relief to offset

[24]In Section 8 we allow the government to fund the bailout expenditure partially through issuing government debt. There we confirm that these crisis dynamics do not depend on the assumption of immediate

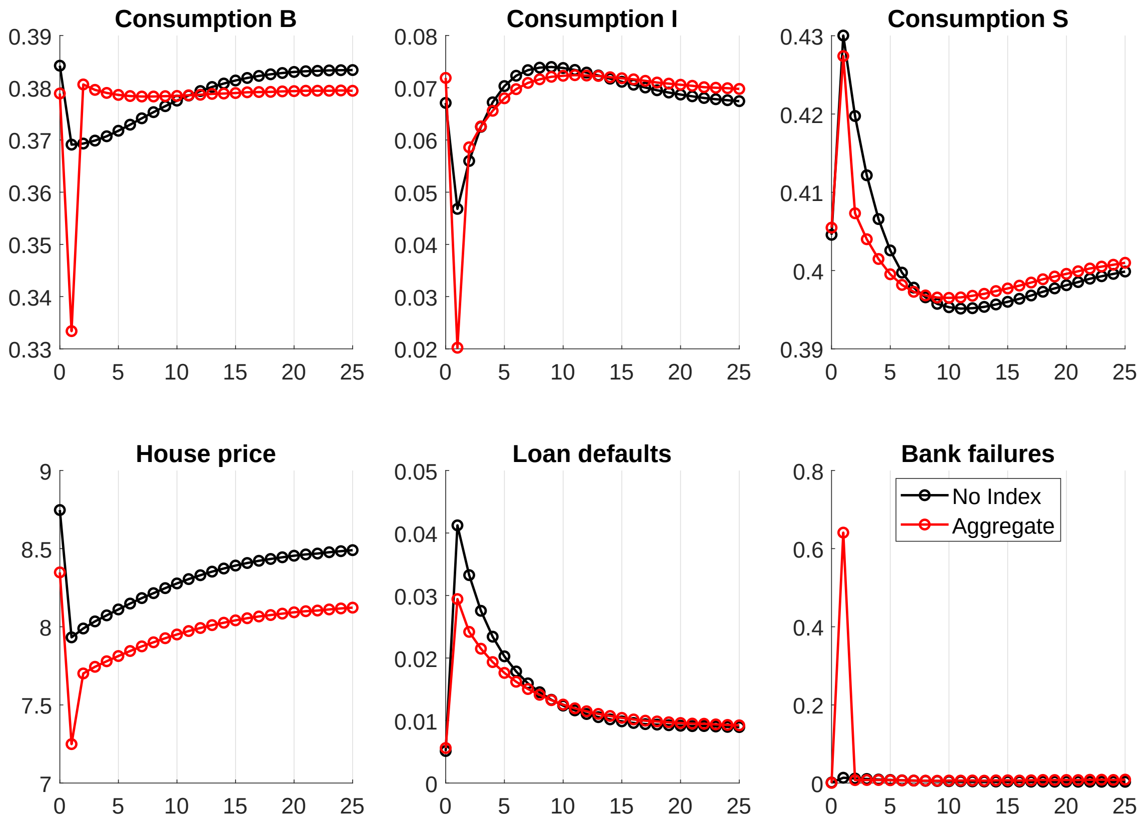
Table 2: Results: Main Indexation Experiments

	No Index	Aggregate	Local	Regional
	Borrower			
1. Housing capital	0.456	0.456	0.462	0.463
2. Refi rate	3.82%	3.77%	3.76%	3.73%
3. Default rate	0.97%	0.98%	0.51%	0.47%
4. Household leverage	64.31%	64.29%	65.71%	65.61%
5. Mortgage debt to income	250.06%	239.42%	267.96%	261.95%
6. Loss-given-default rate	37.31%	35.52%	36.76%	35.75%
7. Loss rate	0.40%	0.40%	0.21%	0.19%
	Intermediary			
8. Bank equity ratio	7.04%	7.14%	7.13%	7.22%
9. Bank default rate	0.30%	1.08%	0.16%	0.40%
10. DWL of bank defaults	0.03%	0.10%	0.02%	0.04%
11. Deposits	2.003	1.882	2.174	2.098
12. Saver mortgage share	14.43%	15.81%	13.30%	14.17%
	Prices			
13. House price	8.533	8.161	8.832	8.616
14. Risk-free rate	0.76%	0.75%	0.77%	0.77%
15. Mortgage rate	1.56%	1.67%	1.37%	1.43%
16. Credit spread	0.80%	0.92%	0.60%	0.66%
17. Mortgage risk prem.	0.40%	0.52%	0.39%	0.46%
	Welfare			
18. Aggregate welfare	0.872	-0.16%	+0.18%	+0.09%
19. CEV welfare	+0.00%	-9.35%	+44.93%	+37.08%
20. Value function, B	0.398	-0.63%	+0.51%	+0.19%
21. Value function, S	0.408	-0.04%	+0.23%	+0.20%
22. Value function, I	0.066	+1.93%	-2.06%	-1.18%
	Consumption and Risk-sharing			
23. Consumption, B	0.382	-0.8%	+0.6%	+0.3%
24. Consumption, S	0.404	+0.0%	+0.1%	+0.2%
25. Consumption, I	0.067	+3.2%	-2.5%	-1.1%
26. Consumption gr vol, B	0.55%	+238.6%	+12.5%	+26.2%
27. Consumption gr vol, S	1.14%	-1.0%	-25.3%	-18.1%
28. Consumption gr vol, I	5.66%	+284.8%	-49.7%	+134.2%
29. Wealth gr vol, I	0.045	+1268.6%	-37.4%	+375.9%
30. log (MU B / MU S) vol	0.026	-0.3%	-9.2%	-35.1%
31. log (MU B / MU I) vol	0.068	+134.2%	-35.1%	+66.1%

taxation, but are a result of the breakdown in mortgage credit.

the instability of intermediaries but only moderate the crisis.

Figure 5: Financial Recessions: Benchmark vs. Aggregate Model



Black: benchmark financial recession, **Red:** aggregate index. financial recession. Responses are plotted in levels.

Aggregate indexation provides a modest reduction in mortgage defaults in the financial recession. Although aggregate indexation protects borrowers from the large fall in national house prices, it is unable to stave off the increase in defaults due to higher idiosyncratic dispersion $\sigma_{\omega,t}$ that accompanies the financial recession. Importantly, aggregate indexation provides equal relief to the hardest-hit and relatively unaffected regions/households alike. This indiscriminately targeted aid limits the policy's effect on the number of foreclosures.

The bottom half of Table 2 compares welfare and consumption outcomes across the different indexation regimes. Aggregate indexation is bad for aggregate welfare. We present two schemes to aggregate the value functions of the three types of agents. The first one simply adds up the value functions, which are expressed in consumption units and already reflect the population mass of each type of agent. This measure shows a 0.16% welfare loss from Aggregate indexation. The second one computes the one-time payments each type of household would be willing to make to transition permanently from the No Index to the alternative indexation economy. Different agents have different valuations for a dollar of consumption since their SDFs differs. We then weigh these consumption equivalent values by the population shares and add up across the types. A positive CEV value indicates that the indexation is a Pareto improvement after transfers. Aggregate indexation results in negative 9.3% CEV measure, implying that agents would need to receive a one-time payment of 9.3% of aggregate consumption in the No-index economy to be willing to switch to Aggregate indexation. Thus, both measures suggest a welfare loss to society.

Underlying the aggregate welfare result are interesting distributional differences. Borrowers are made worse off (row 20). Their consumption is lower (row 23) and becomes much more volatile (row 26). The increased financial fragility from Aggregate indexation results in incredibly volatile intermediary wealth (W^I growth volatility goes up 1268.6%). Intermediary consumption growth volatility goes up 284.8%, and intermediary consumption falls sharply in a financial recession. These results point to a deterioration in risk sharing between borrowers and intermediaries, further evidenced by a rise of 134.2% in the volatility of the log marginal utility ratio between these types (row 31). Despite the rise in consumption volatility, intermediaries are made better off (row 22). Aggregate indexation raises the average credit spread, mortgage risk premium, and REO returns, and hence the profitability of intermediation. Also, aggregate indexation increases the value of banks' default option, allowing for very high consumption in good times but limited downside in bad times. Banks thrive when financial turmoil is large. Finally, savers' welfare falls modestly (row 21) due to lower consumption in financial recessions, which are high marginal utility times. The latter is in part due to higher taxes that need to be raised to cover losses from bank bailouts. Since savers are more patient, they have a larger shadow value of consumption; their welfare loss weighs heavily in the CEV welfare measure. All told, insuring borrower exposure to aggregate house price risk paradoxically hurts the borrowers it was meant to help, hurts savers, but benefits financial intermediaries.

6.2 Local Indexation

Next, we turn to the Local economy ($\iota_p = 0, \iota_\omega = 1$), which indexes only to the local component of house values. In practice, such a contract would be implemented by subtracting an aggregate house price index from regional indexes, and then indexing the debt of local borrowers to the local residual. For example, during the Great Recession house prices fell substantially more in Las Vegas than in Boston. Local indexation would have implied a reduction in mortgage debt for Las Vegas borrowers, but an increase in debt for Boston borrowers. While such indexation is unlikely to ever see the light of day, it is an important building block for Regional indexation.

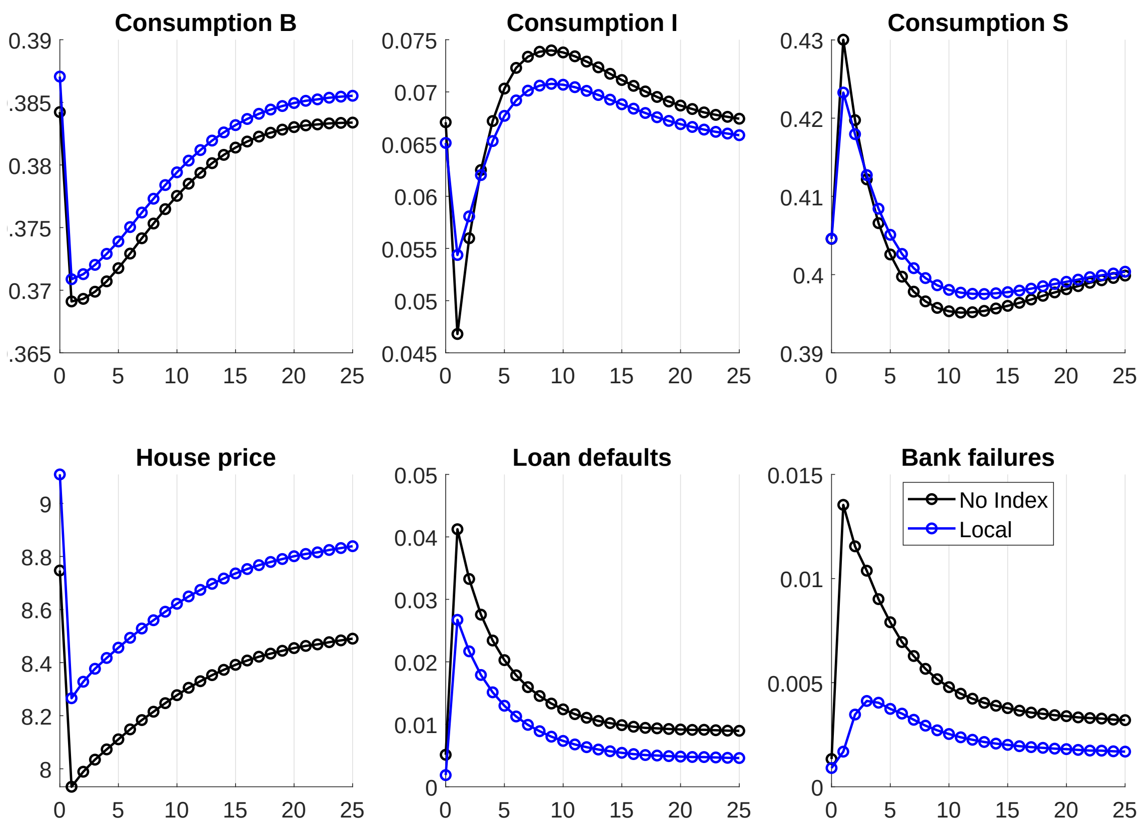
In sharp contrast to Aggregate indexation, indexing mortgage debt to relative local house prices stabilizes the financial sector while substantially reducing the frequency of borrower defaults. Figure 6 compares financial recessions in the No-Index and Local models. Although borrowers must absorb a similar fall in aggregate house prices as in the baseline, local indexation is still largely successful at reducing foreclosures. It sends targeted debt relief to households in areas where house prices fall the most.[25] Unlike in the aggregate indexation case, the reduction in defaults under local indexation is not accompanied by large financial sector losses, since the diversifiable local shocks wash out. As a result, the rate of bank failures in a financial crisis is markedly *lower* under local indexation, a sign of improved financial stability. Savers hold a smaller share of mortgage debt directly when intermediaries are more stable.

Turning to unconditional moments in the third column of Table 2, we observe that the average mortgage default rate falls precipitously, with a reduction of nearly half relative to the benchmark. While aggregate and local indexation are roughly equally effective at reducing default in a financial crisis, when default is largely driven by aggregate house prices, local indexation is much more effective than aggregate indexation in normal times, when default is primarily driven by local and idiosyncratic shocks. Facing less default risk, banks lower mortgage interest rates. This pushes up house values and supports increased household borrowing. The higher average stock of mortgage debt is financed with a larger deposit base. While banks react to this reduced risk by holding as little capital as allowed, the required minimum is sufficient to ensure a large decrease in the rate of bank failures. The risk-free interest rate rises slightly as the supply of deposits expands to meet the demands of a larger intermediation sector. Overall, the banking system is both safer and larger in the Local economy, but it receives less compensation for risk on a per-loan basis.

The welfare effects from local indexation are the reverse of those from aggregate indexation. Local indexation is good for aggregate welfare according to both measures. The population-weighted welfare function increases by 0.183%, and agents would be willing to pay 44.9% of aggregate consumption to transition to Local indexation according to the CEV criterion. Borrowers and savers gain while intermediaries lose. Risk sharing in the economy improves dramatically, as the volatility of marginal utility ratios between groups

[25] For intuition, recall that the average borrower in the model, similar to the data, has typical leverage around 65%. Thus, the typical borrower could absorb a very large fall in aggregate house prices (on the order of the 2008 housing crash) and still remain above water. Instead, the typical defaulting borrower must *also* receive an adverse local or idiosyncratic shock. Effectively indexing against these shocks is therefore a potent force against default, even during an aggregate house price decline.

Figure 6: Financial Recessions: Benchmark vs. Local Model



Black: benchmark financial recession, **Blue:** local index. financial recession. Responses are plotted in levels.

falls, especially between borrowers and intermediaries. Savers and intermediaries also see large reductions in consumption growth volatility, while borrowers experience increased volatility – albeit from a low level – due to larger housing and mortgage positions.[26]

In sum, indexation to local house price shocks is highly effective at reducing the risk of foreclosures and financial fragility. More intermediation ensues, which makes both borrowers and savers richer. However, the increased safety makes banking less profitable.

6.3 Regional Indexation

The fourth column of Table 2 shows results for the Regional model, which indexes mortgage principal and interest payments to both aggregate and local house price variation. Unsurprisingly, the simulation means in this column mostly lie between the Aggregate and Local

[26]The smaller changes in intermediary and depositor consumption during crises (top row of Figure 6) underscore this point. Depositors earn higher interest rates under this system, while borrowers pay lower rates on their mortgages, helping to boost the consumption of each group. In contrast, intermediary households' mean consumption falls by 2.5% as dividends from REO firms and banks decline.

cases in columns 2 and 3. Pairing local and aggregate indexation decreases the bank default rate in the Regional model relative to the Aggregate model. But the destabilizing effect of aggregate indexation is still enough to increase bank defaults relative to the No-Index baseline. The high consumption and wealth growth volatilities of the intermediary are further signs of financial instability. The high degree of indexation in this economy strongly reduces the incentives to default, leading to the lowest borrower default rates among the four models. Aggregate welfare is 0.09% higher in the Regional model than in the No Index model according to the population-weighted measure, and the willingness to pay is 37.1% of according to the CEV measure.

7 Liquidity Defaults

So far, we have assumed that mortgage borrowers default when it is optimal to do so: when the market value of their mortgage exceeds the market value of the housing collateral. This section considers a model extension where defaults are driven by both liquidity concerns (the need to stop making mortgage payments) and strategic motives. A recent empirical literature has argued that liquidity defaults were prominent during the Great Recession (See e.g., Bhutta, Shan, and Dokko, 2010; Gerardi, Herkenhoff, Ohanian, and Willen, 2017).

To incorporate liquidity defaults, we assume that fraction θ of borrowers are hit by liquidity shocks each period. These shocks stand in for adverse idiosyncratic income or expenditure shocks (unemployment, disability, divorce, etc). Borrowers hit with the shock go into foreclosure if the book value of their debt, $M_t^B + A_t^B$, exceeds a fraction Ξ of the market value of their house.[27] For strategic default, we introduce an extra cost to the borrower of losing his or her home, equal to fraction η^B of the home's value. This allows us to capture the observation that borrowers do not tend to strategically default until they are well under water. We set $\theta = 0.18$ to generate a 93% share of liquidity-driven defaults. We set $\Xi = .9$ to capture that a substantial fraction of liquidity defaults are above-water defaults. We further set $\eta^B = 0.05$, implying that, on average, the cost of strategic default amounts to 57% of annual consumption. These three parameters deliver three moments matching the evidence in Ganong and Noel (2019b). We need to adjust other parameters to match the average default rate and the level of housing wealth relative to income in the model with liquidity defaults. We provide details in Appendix C.3.

Table 3 summarizes the unconditional moments. The model with liquidity default is

[27] Borrowers below the threshold do not default. Since we assume perfect consumption risk sharing among borrowers, liquidity shocks that do not trigger default have no consumption consequences in our framework.

similar to our baseline setup with only strategic default along many dimensions. The welfare comparison between the No-Index, Aggregate, Local, and Regional economies is both qualitatively and quantitatively unaffected. This near equivalence of results despite very different motives for default occurs for two reasons. First, our extended model reflects a "double-trigger" view of default. Even though a liquidity shock is usually a necessary condition for default, it is not sufficient. Only liquidity-shocked borrowers above the leverage threshold end up defaulting. As a result, default remains highly dependent on house prices and leverage, consistent with the evidence in Section 2. Second, the leverage threshold for strategic default is optimally chosen, while the liquidity default threshold reflects borrowers' need to alleviate a liquidity crunch. As a result, some borrowers hit by liquidity shocks default despite positive home equity, leading to higher recovery values for banks. This offsets bank losses from strategic defaulters that have much lower home equity in the extended model with the strategic default penalty. The net effect are slightly smaller losses for banks given the same overall default rate.

8 Extensions

We consider several extensions, with details relegated to the Online Appendix.

8.1 Interest vs. Principal Indexation

So far, our indexation applied both to interest payments and to principal. However, a number of the contract proposals in the literature envision indexing interest payments only. These proposals are motivated by empirical work by Fuster and Willen (2015) and Di Maggio et al. (2017) who suggest that households respond strongly to interest payment adjustments, and Ganong and Noel (2019a) who show that households barely respond to principal adjustments, at least when the latter leave them underwater. We run experiments in which either interest or principal payments, but not both, are indexed to house prices. The corresponding default thresholds are derived in Appendix C.1.

The first four columns of Table 4 contrast the No Index and Regional models with Reg-IO and Reg-PO specifications that index only interest and principal payments to regional house prices, respectively. The main result is that indexing interest only greatly dilutes the effects of indexation, reducing its ability to mitigate borrower defaults, while indexing principal only delivers results very similar to full indexation. Quantitatively, the Reg-IO model delivers a borrower default rate of 0.82%, much higher than the Regional model's 0.47%, and close to the 0.97% of the No Index model. The Reg-PO model's 0.51% default rate is nearly as low as the Regional model's default rate.

This result is perhaps surprising given that our baseline model mortgage payments are on average 75% interest and 25% principal, closely matching reality. The key to this result is that our model mortgages are prepayable, and our model borrowers (realistically) choose to refinance or renew them every six to seven years. But while a lower principal balance provides equity extraction opportunities at this time, the interest rate is reset upon receiving a new loan, wiping out further gains from interest indexation. As a result, the temporary gains from interest forgiveness under IO indexation are valued less than the permanent gains from principal forgiveness under PO indexation, leading to a smaller overall impact. By the same logic, forgiving interest payments is less costly to intermediaries than forgiving principal, mitigating their losses during housing declines, and avoiding an increase in financial fragility and bank defaults.

We repeat this exercise in our liquidity default model (columns 3 and 4 of Table 5), and find that this contrast between IO and PO indexation is even stronger: now the Reg-IO model delivers a borrower default rate indistinguishable from the No Index model (1.11% vs. 1.12%), while the Reg-PO and Regional models' rates are nearly equal (0.68% vs. 0.67%). Under strategic default, forgiving interest payments incentivizes borrowers not to default by lowering the costs of repayment. In contrast, borrowers experiencing a liquidity event must either sell if their principal balance is low enough to repay their loan, or default otherwise. Since this decision does not depend on interest rate incentives, interest indexation has no direct influence on liquidity default.[28]

8.2 Asymmetric Indexation

Some real-world SAM proposals envision reducing mortgage payments when house prices fall but not increasing payments when prices rise. We now study such asymmetric contracts. We assume indexation to both aggregate and local house price components (Regional model), but cap the maximum upward indexation in both dimensions. With asymmetric indexation, our assumption of i.i.d. house quality shocks $\omega_{i,t}$ is no longer equivalent to more realistic persistent $\omega_{i,t}$ processes. To address this, we model the $\omega^L_{i,t}$ and $\omega^U_{i,t}$ shocks as AR(1) processes for the results below. Appendix C.2 provides details on this extension and the corresponding optimality conditions.

Column 5 of Table 4 presents the results for the Reg-Asym case, demonstrating that

[28]In fact, most liquidity defaulters would prefer keeping their mortgage to defaulting with or without interest forgiveness, but simply cannot afford to.

asymmetric indexation substantially alters the mortgage landscape. Banks now expect to take losses on average from indexation, since the debt relief they offer on the downside is no longer compensated by higher debt payments when house prices increase. As a result, banks set much higher mortgage rates ex-ante, 3.24% higher per year than without indexation, to compensate for the asymmetric transfers to the households. At the aggregate, this has an effect similar to shortening the mortgage amortization schedule (lower δ), since borrowers make higher coupon payments in exchange for a much larger effective principal reduction each period, albeit one occurring largely through indexation rather than explicit principal payments. House prices are lower, reflecting the lower collateral value of housing under this more front-loaded contract. Lower house prices imply lower mortgage balances, lower deposits, and a smaller financial sector overall. Savers intermediate a larger share of mortgage debt.

Although borrowers partially compensate for the higher mortgage rates by increasing the refinancing rate, the faster effective amortization of these loans dominates, reducing household leverage. Lower leverage in turn virtually eliminates foreclosures, since it now takes much larger shocks to push borrowers underwater. Nonetheless, financial fragility is massively increased under Reg-Asym. When indexation is symmetric, the large losses the financial sector suffers when house prices fall are partially offset by expected gains from indexation as house prices rise. Asymmetric indexation removes this mitigating force, leading to an extremely high bank failure rate of 0.83%, more than twice as high as in the symmetric Regional model.

Turning to total welfare, the gain of +0.41% is the highest among all contracts we consider. These gains are driven by a fall in deadweight losses from foreclosure, increasing aggregate consumption, and overpowering the deterioration of risk sharing observed from this model's high volatilities of borrower consumption and intermediary wealth. However, we note that since these foreclosure reductions occur largely through lower household leverage, other measures to reduce household leverage (e.g., lowering maximum LTVs) might attain the same benefits without increasing financial fragility. Although borrowers must finance more bailouts under asymmetric indexation, they are more than compensated by house price gains in good times which they retain under asymmetric indexation.

Asymmetric IO. Column 6 of Table 4 presents the asymmetric indexation of interest payments only (Asym-IO), leaving the principal balance and payments unindexed. Similar to the findings in Section 8.1, indexing interest only dilutes the positive welfare effects of the Reg-Asym contract. The Asym-IO model has a higher foreclosure rate (0.54% vs. 0.13%) and a lower bank failure rate (0.29% vs. 0.83%). Household leverage again falls, in part for the same reasons as in the Reg-Asym case, but also due to a different and novel force. Because interest is lowered over time through indexation, but principal is not, the effective interest rates on existing loans tends to be lower than the interest rates on new loans. Borrowers respond by refinancing their loans less often, causing longer periods between equity extractions, and reducing average leverage. Overall, the Asym-IO contract is much less disruptive than the full Reg-Asym contract, delivering a substantial reduction in foreclosures while slightly reducing bank failures and improving measures of risk sharing (rows 26-31).

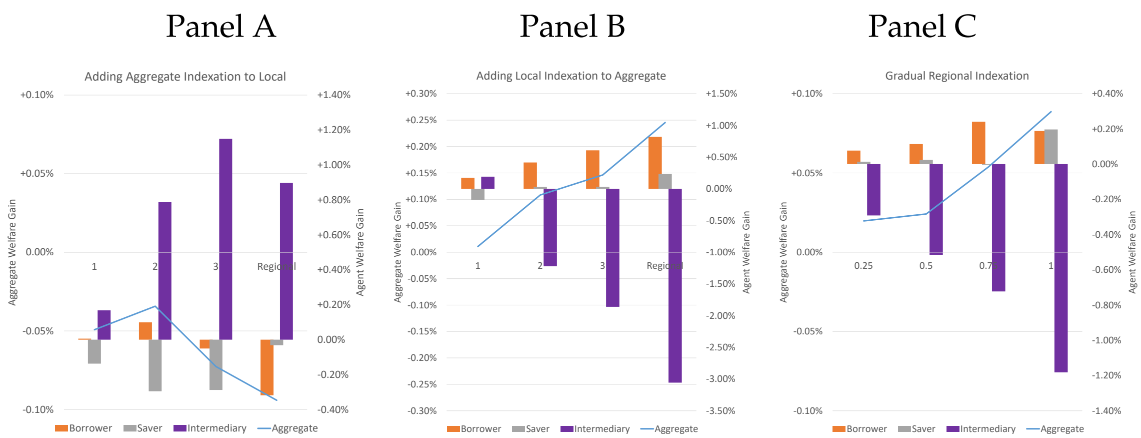
Tail Indexation. The final contract type we consider is tail indexation (Reg-Tail), in which the borrower is responsible for the first 10% of regional price declines and the lender fully indexes any decline beyond that threshold. This scheme is similar to the Reg-Asym scheme, except that indexation kicks in at a positive level of losses instead of at zero losses. The resulting economy features a foreclosure rate of 0.34%, and a bank failure rate of 0.38%, both of which are improvements over the Regional model. This superior performance is due to the more efficient intervention of the Reg-Tail model, which only provides enough relief to prevent households from becoming underwater, in contrast to the Reg-Asym model which seeks to insure *all* house price declines. Avoiding excessive indexation allows for effective reduction in the default rate without overburdening the financial sector, limiting the increase in financial fragility.

Liquidity Default. The last three columns of Table 5 display the corresponding statistics for the asymmetric indexation models with liquidity default. Our findings on asymmetric indexation are largely unchanged with the primary exception of the Asym-IO model, since, as mentioned in Section 8.1, indexing interest only is much less effective at preventing liquidity defaults than strategic defaults in our framework.

8.3 Partial Indexation

So far, we have considered full aggregate indexation ($\iota_p = 1$), full local indexation ($\iota_\omega = 1$), or both. But might intermediate levels of indexation be optimal? Panel A of Figure 7 gradually adds aggregate indexation to an economy that already has full local indexation. Panel B gradually adds local indexation to an economy with full aggregate indexation. Panel C gradually adds both types of indexation in lock-step. The effect on the value function on each type of agent is indicated by bars and measured against the right axis, while the population-weighted aggregate welfare measure is indicated by a solid line plotted against the left axis. Each set of bars increases indexation by 25%. The end point in each panel is the same Regional economy but the starting point and hence the welfare changes are different.

Figure 7: Partial Indexation



Even only adding a small amount of aggregate indexation to an economy that already has full local indexation is not good for welfare. The gains to the borrowers and the intermediaries do not outweigh the losses to the savers. The local indexation provides a good amount of financial sector stability. Adding 25% or 50% aggregate indexation lowers borrower and bank default rates. It lowers mortgage rates and increases mortgage debt and house prices. When aggregate indexation becomes greater than 50%, financial fragility increases and borrowers begin to lose relative to the world with only local indexation.

Panel B shows that adding local indexation to an economy that has aggregate indexation monotonically increases welfare. Borrowers and savers gradually gain by more while intermediaries gradually lose. The same result holds in Panel C for an economy that gradually implements Regional indexation, starting from No Indexation.

8.4 Tighter Bank Leverage Constraints

A possible response to the destabilizing effects resulting from Aggregate indexation could be to tighten bank capital requirements. Solving the model with a minimum bank equity capital ratio of 10% rather than 7%, we find indeed that tighter leverage reduces bank failure rates substantially from 1.08% to 0.22%, but it does not lower the welfare losses from Aggregate indexation. The reason is that tighter macro-prudential policy shrinks the banking sector. Aggregate indexation shrinks deposits by 6% when banks minimum capital is 7% but by more than twice that (13%) when bank capital requirements are 10%. Reduced intermediation capacity results in higher credit spreads and larger welfare gains for banks.

But savers' and borrowers' welfare losses are substantially larger, due to the larger fall in deposits and the higher mortgage rates, respectively. Tighter bank capital requirements cannot rescue Aggregate indexation.

8.5 Risk Absorption Capacity

We perform two exercises to analyze the sensitivity of the results to the intermediary's risk absorption capacity.

Intermediary population share. In a first exercise, we change the population share of intermediaries from 5% to 3%, 4%, 6%, or 10%.[29] Regional indexation delivers an aggregate population-weighted welfare gain of 0.09% at the benchmark 5% population share. The welfare gain is increasing in the intermediaries' population share, from -0.77% at a 3% share to +0.18% at a 10% share (Appendix Table C.1). With lower intermediary risk absorption capacity, there is more financial fragility. Mortgage defaults are slightly higher but bank failure rates are substantially higher. Bank default rates are 7.5 times higher for the 3% than for the 10% economy. The 3% economy has much higher credit spreads and mortgage risk premia than the 10% economy, resulting in lower mortgage debt and house prices, and lower borrower welfare. In sum, we are getting the intuitive result that the welfare effects of regional indexation depend on the risk absorption capacity of the intermediary sector. Regional indexation achieves positive welfare effects only when intermediaries have sufficiently large risk absorption capacity.

Saver holdings of mortgages. In a second exercise, we switch off the ability of savers to directly hold mortgage debt by increasing the cost parameter φ_0 to a very high value. Intermediaries are responsible for all mortgage market intermediation. They choose to hold more equity capital in this economy, not only in dollars but also as a ratio of bank assets. They face less financial fragility as a result; baseline bank default rates are only 0.10% versus 0.30% in the model with saver holdings of mortgage debt. Aggregate indexation is better for overall welfare and local indexation is slightly worse in the model without saver holdings. Regional indexation, which combines both, results in the same quantitative welfare gain in

[29]To identify the corresponding income shares for the intermediaries in these four model variants, we first find a new risky asset share cutoff in the SCF data that delivers the desired population share. The intermediary income share is then the observed share of income in the SCF for the resulting group of households whose risky asset share is above the cutoff. The saver income share is the income share of the complement group of households. The population share share of savers changes in the opposite direction and by the same absolute value as that of the intermediary households.

the models with and without saver holdings.

The key difference between both models is that risk-free rates fall by more during a financial recession in the economy without direct saver holdings. This benefits banks because it aids their subsequent recapitalization. The intuition for this effect is as follows. If savers cannot directly hold mortgages, their only store of wealth are bank deposits. During recessions, banks significantly shrink their deposit issuance, which is an inward shift of the demand curve in the deposit market. As a result, the deposit interest rate drops sharply. When savers can also directly hold mortgages, their supply of deposits to banks effectively becomes more elastic. Therefore, the interest rate falls by less in housing recessions when savers can directly invest in mortgages. Since deposit interest rates fall by less, banks earn lower returns during the transition out of a housing recession, causing a more sluggish recovery. In sum, our main indexation results on the merits of aggregate and local indexation are slightly *amplified* when savers hold mortgage debt directly.

8.6 Government Debt

Our baseline model assumes that the government raises lump-sum taxes to fully pay for bank bailouts within each period. When a large fraction of banks fail, taxes required to fund the bailout reduce consumption, most notably in the aggregate indexation model (Figure 5). This immediate tax burden might be smaller if the government financed bailouts with debt, potentially reducing the severity of financial recessions. To test the sensitivity of crisis dynamics to different taxation regimes, we solve the Aggregate model with some tax smoothing. Each period, the government uses taxes to pay 80% of its outstanding liabilities (past debt plus expenses for current bailouts), with the remainder funded by new debt.

Appendix Figure C.1 compares crisis dynamics in the Aggregate indexation model with government debt ($\tau_L = 0.8$) to the Aggregate model with immediate taxation ($\tau_L = 1$). Borrower consumption falls by slightly less on impact, as the tax burden is postponed further into the future. However, saver consumption is substantially reduced, as savers must purchase the government debt that funds the bailout. To induce the savers to absorb this debt, the real risk-free interest rate, which is both the deposit rate and the yield on government debt, needs to increase compared to the immediate-taxation model. At this higher real rate, banks issue fewer deposits as government safe asset provision crowds out private safe asset production (Azzimonti and Yared, 2018). The higher real rate increases banks' funding cost and compresses mortgage spreads, depressing intermediary consumption. Banks respond to the lower supply of deposits by cutting their lending more sharply, reducing the availability of mortgage credit to borrowers, leading to a sharper drop in house prices. At the same time, higher funding costs reduce bank net worth, increasing the rate of bank failures. Maybe surprisingly, financial fragility increases in the economy with government debt. The comparison demonstrates that the severe crisis dynamics with Aggregate indexation are not an artifact of the assumption of instantaneous taxation. Instead, the primary driver of the steep drop in house prices is the sharp contraction in the size of the financial sector. This contraction is only amplified when bailouts are funded through government debt since the higher cost of deposit funding causes a larger decline in lending.

9 Conclusion

Redesigning the mortgage market through product innovation may allow an economy to avoid a severe foreclosure crisis like the one that hit the U.S. economy in 2008-2010. To this end, we study the implications of indexing mortgage payments to house prices in a general equilibrium model with incomplete risk-sharing, costly default, and a rich intermediation sector. A key finding is that indexing mortgage debt to aggregate house prices may increase financial fragility. Inflicting large losses on highly-levered lenders in bad states of the world can cause systemic risk (high bank failure rates), costly taxpayer-financed bailouts, larger house price declines, and higher risk premia on mortgages, all of which ultimately hurt the borrowers the indexation was intended to help. Moreover, aggregate indexation redistributes wealth from borrowers and savers towards bank owners, since a more fragile banking business also is a more profitable banking business. In sharp contrast, indexation of cross-sectional local house price risk is highly effective at reducing mortgage defaults and financial fragility. It increases welfare for borrowers and savers, while reducing it for intermediaries, as mortgage banking becomes safer but less profitable.

Our results show that mortgage indexation in a world where intermediaries have limited liability and risk absorption capacity has important general equilibrium effects. Although potential benefits exist, indexation schemes must be designed carefully to attain them. We conclude that less invasive approaches such as our tail indexation model that concentrate their effects on limiting the severe losses that cause defaults, while leaving mortgages unindexed when they appear far from default, could provide substantial benefits with minimal disruption to financial stability.

The framework proposed in this paper could be extended in several directions to allow for other costs and benefits of mortgage indexation. Considering imperfectly insurable idiosyncratic labor income risk and its interaction with mortgage indexation would be a fruitful extension. We conjecture that adding uninsurable individual income risk to our setup would further strengthen the benefits of local indexation, since indexation to local house price shocks would provide insurance against local labor market risk. A second promising extension could consider an economy where indexed and non-indexed contracts co-exist, with the share of indexed contracts endogenously varying with the state of the economy. To the extent that these shares covary with the health of the financial sector, this might have important implications for the costs of indexation during a crisis.

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Table 3: Results: Liquidity Default

	Baseline	Model with Liquidity Defaults			
	No Index	**No Index**	**Aggregate**	**Local**	**Regional**
			Borrower		
1. Housing capital	0.456	0.450	0.451	0.457	0.457
2. Refi rate	3.82%	3.76%	3.76%	3.73%	3.74%
3. Default rate	0.97%	1.12%	1.06%	0.71%	0.67%
Share liq. defaults	0.00%	94.81%	94.19%	97.77%	97.62%
4. Household leverage	64.31%	65.67%	65.62%	66.30%	66.45%
5. Mortgage debt to income	250.06%	285.77%	287.05%	283.56%	292.52%
6. Loss-given-default rate	37.31%	25.76%	24.99%	23.95%	24.60%
7. Loss rate	0.40%	0.34%	0.30%	0.20%	0.18%
			Intermediary		
8. Bank equity ratio	7.04%	7.17%	7.77%	6.98%	7.76%
9. Bank default rate	0.30%	0.11%	0.36%	0.12%	0.07%
10. DWL of bank defaults	0.03%	0.01%	0.04%	0.01%	0.01%
11. Deposits	2.003	2.346	2.334	2.311	2.400
12. Saver mortgage share	14.43%	12.29%	12.47%	12.99%	11.75%
			Prices		
13. House price	8.533	9.675	9.691	9.368	9.621
14. Risk-free rate	0.76%	0.77%	0.75%	0.71%	0.78%
15. Mortgage rate	1.56%	1.50%	1.54%	1.34%	1.36%
16. Credit spread	0.80%	0.73%	0.78%	0.63%	0.58%
17. Mortgage risk prem.	0.40%	0.38%	0.48%	0.43%	0.39%
			Welfare		
18. Aggregate welfare	0.872	0.869	-0.04%	+0.22%	+0.14%
19. CEV welfare	+0.00%	+0.00%	-16.08%	+2.67%	+4.41%
20. Value function, B	0.398	0.392	-0.25%	+1.16%	+0.69%
21. Value function, S	0.408	0.408	-0.08%	+0.00%	+0.02%
22. Value function, I	0.066	0.069	+1.41%	-3.77%	-2.30%
			Consumption and Risk-sharing		
23. Consumption, B	0.382	0.377	-0.2%	+1.3%	+0.9%
24. Consumption, S	0.404	0.405	-0.2%	-0.2%	+0.0%
25. Consumption, I	0.067	0.071	+2.3%	-4.1%	-2.8%
26. Consumption gr vol, B	0.55%	0.42%	+78.8%	-11.5%	-31.4%
27. Consumption gr vol, S	1.14%	1.03%	-5.9%	-31.5%	-35.6%
28. Consumption gr vol, I	5.66%	5.38%	+160.3%	-39.2%	+9.2%
29. Wealth gr vol, I	0.045	0.034	+730.5%	-42.1%	+141.4%
30. log (MU B / MU S) vol	0.026	0.022	-22.7%	-24.8%	-49.8%
31. log (MU B / MU I) vol	0.068	0.080	+47.2%	-27.9%	-12.7%

Table 4: Results: Alternative Indexation Schemes

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO	Reg-Tail
	Borrower						
1. Housing capital	0.456	0.463	0.458	0.462	0.468	0.461	0.465
2. Refi rate	3.82%	3.73%	3.72%	3.76%	4.40%	3.54%	4.28%
3. Default rate	0.97%	0.47%	0.82%	0.51%	0.13%	0.54%	0.34%
4. Household leverage	64.31%	65.61%	65.19%	65.52%	58.47%	62.65%	60.03%
5. Mortgage debt to income	250.06%	261.95%	260.03%	265.09%	230.12%	257.11%	239.16%
6. Loss-given-default rate	37.31%	35.75%	38.88%	38.36%	33.71%	29.23%	36.63%
7. Loss rate	0.40%	0.19%	0.27%	0.22%	0.91%	0.35%	0.90%
	Intermediary						
8. Bank equity ratio	7.04%	7.22%	7.04%	7.13%	6.95%	6.81%	6.98%
9. Bank default rate	0.30%	0.40%	0.24%	0.32%	0.83%	0.29%	0.38%
10. DWL of bank defaults	0.03%	0.04%	0.02%	0.03%	0.07%	0.03%	0.03%
11. Deposits	2.003	2.098	2.090	2.135	1.803	1.991	1.898
12. Saver mortgage share	14.43%	14.17%	13.87%	13.82%	16.43%	14.09%	15.24%
	Prices						
13. House price	8.533	8.616	8.686	8.742	8.409	8.571	8.566
14. Risk-free rate	0.76%	0.77%	0.76%	0.75%	0.76%	0.76%	0.77%
15. Mortgage rate	1.56%	1.43%	1.43%	1.40%	2.37%	1.61%	2.10%
16. Credit spread	0.80%	0.66%	0.68%	0.65%	1.61%	0.84%	1.33%
17. Mortgage risk prem.	0.40%	0.46%	0.39%	0.43%	0.50%	0.41%	0.40%
	Welfare						
18. Aggregate welfare	0.872	+0.09%	+0.03%	+0.10%	+0.41%	+0.08%	+0.36%
19. CEV welfare	+0.00%	+37.08%	+7.74%	+14.77%	-13.52%	+3.01%	-4.94%
20. Value function, B	0.398	+0.19%	+0.16%	+0.32%	+1.75%	+0.56%	+1.54%
21. Value function, S	0.408	+0.20%	+0.04%	+0.08%	-0.10%	+0.01%	-0.05%
22. Value function, I	0.066	-1.18%	-0.78%	-1.14%	-4.49%	-2.38%	-4.25%
	Consumption and Risk-sharing						
23. Consumption, B	0.382	+0.3%	+0.2%	+0.5%	+2.1%	+0.7%	+1.9%
24. Consumption, S	0.404	+0.2%	+0.0%	+0.1%	-0.0%	+0.0%	-0.1%
25. Consumption, I	0.067	-1.1%	-0.7%	-1.4%	-5.2%	-2.9%	-5.4%
26. Consumption gr vol, B	0.55%	+26.2%	-13.3%	-24.2%	+57.3%	-1.0%	+15.6%
27. Consumption gr vol, S	1.14%	-18.1%	-9.9%	-14.8%	-20.9%	-19.7%	-8.8%
28. Consumption gr vol, I	5.66%	+134.2%	-6.4%	+96.9%	+128.6%	-25.7%	-26.6%
29. Wealth gr vol, I	0.045	+375.9%	-12.6%	+272.4%	+687.5%	-8.1%	+21.4%
30. log (MU B / MU S) vol	0.026	-35.1%	-9.6%	-37.5%	-13.2%	-12.6%	+8.6%
31. log (MU B / MU I) vol	0.068	+66.1%	-6.9%	+49.0%	+42.8%	-24.3%	-44.7%

Table 5: Results: Liquidity Default, Alternative Indexation Schemes

	No Index	**Regional**	**Reg-IO**	**Reg-PO**	**Reg-Asym**	**Asym-IO**	**Reg-Tail**
				Borrower			
1. Housing capital	0.450	0.457	0.450	0.457	0.467	0.447	0.460
2. Refi rate	3.76%	3.74%	3.76%	3.74%	4.33%	3.53%	4.09%
3. Default rate	1.12%	0.67%	1.11%	0.68%	0.15%	1.12%	0.58%
4. Household leverage	65.67%	66.45%	66.15%	66.40%	59.75%	64.44%	62.82%
5. Mortgage debt to income	285.77%	292.52%	292.27%	292.28%	217.57%	285.73%	256.31%
6. Loss-given-default rate	25.76%	24.60%	26.01%	24.80%	20.96%	15.96%	24.80%
7. Loss rate	0.34%	0.18%	0.31%	0.19%	0.74%	0.35%	0.59%
				Intermediary			
8. Bank equity ratio	7.17%	7.76%	7.22%	7.70%	0.00%	0.00%	0.00%
9. Bank default rate	0.11%	0.07%	0.09%	0.07%	0.89%	0.13%	0.24%
10. DWL of bank defaults	0.01%	0.01%	0.01%	0.01%	0.07%	0.01%	0.02%
11. Deposits	2.346	2.400	2.414	2.401	1.688	2.285	2.061
12. Saver mortgage share	12.29%	11.75%	12.08%	11.61%	17.26%	12.34%	14.08%
				Prices			
13. House price	9.675	9.621	9.857	9.616	7.792	9.654	8.873
14. Risk-free rate	0.77%	0.78%	0.76%	0.78%	0.77%	0.77%	0.77%
15. Mortgage rate	1.50%	1.36%	1.46%	1.36%	2.21%	1.59%	1.77%
16. Credit spread	0.73%	0.58%	0.70%	0.58%	1.43%	0.82%	1.00%
17. Mortgage risk prem.	0.38%	0.39%	0.38%	0.39%	0.50%	0.39%	0.40%
				Welfare			
18. Aggregate welfare	0.869	+0.14%	+0.02%	+0.14%	+0.46%	-0.09%	+0.32%
19. CEV welfare	+0.00%	+4.41%	-1.07%	+5.67%	-51.83%	-1.60%	-19.92%
20. Value function, B	0.392	+0.69%	+0.00%	+0.71%	+2.92%	-0.32%	+1.85%
21. Value function, S	0.408	+0.02%	-0.01%	+0.03%	-0.30%	+0.00%	-0.12%
22. Value function, I	0.069	-2.30%	+0.32%	-2.45%	-9.06%	+0.61%	-5.79%
				Consumption and Risk-sharing			
23. Consumption, B	0.377	+0.9%	-0.1%	+0.9%	+3.6%	-0.4%	+2.3%
24. Consumption, S	0.405	+0.0%	-0.0%	+0.0%	-0.3%	+0.0%	-0.1%
25. Consumption, I	0.071	-2.8%	+0.5%	-3.0%	-11.0%	+0.9%	-7.3%
26. Consumption gr vol, B	0.42%	-31.4%	-14.8%	-38.8%	+88.2%	-15.4%	+8.6%
27. Consumption gr vol, S	1.03%	-35.6%	+0.5%	-39.4%	-13.3%	-3.2%	-10.3%
28. Consumption gr vol, I	5.38%	+9.2%	+12.6%	-4.9%	+120.5%	+9.8%	-18.5%
29. Wealth gr vol, I	0.034	+141.4%	+37.8%	+99.8%	+837.4%	+26.5%	+5.1%
30. log (MU B / MU S) vol	0.022	-49.8%	-8.5%	-55.2%	-8.5%	-15.5%	+0.3%
31. log (MU B / MU I) vol	0.080	-12.7%	+1.9%	-21.7%	+12.5%	+7.4%	-30.8%

A Model Derivations

A.1 Stochastic Discount Factors

In our incomplete markets economy, we can construct a separate stochastic discount factor for each representative household, $j = B, I, S$.

Denote the certainty equivalent of future utility of type j as:

$$CE_t^j = \mathbb{E}_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}},$$

where utility U_t^j is defined in equation (2).

The stochastic discount factor of agent j is then defined as

$$\Lambda_{t+1}^j = \beta_j \left(\frac{U_{t+1}^j}{CE_t^j} \right)^{1/\psi - \gamma} \left(\frac{u_{t+1}^j}{u_t^j} \right)^{-1/\psi} \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-1}, \tag{38}$$

where $u_t^j = (C_t^j)^{1-\xi_t} (H_t^j)^{\xi_t}$, the standard definition with Epstein-Zin preferences.

A.2 Aggregation Across Vintages

This appendix shows that a portfolio of long-term fixed-rate mortgages issued in different periods (vintages) at vintage-specific mortgage rates can be completely summarized by two state variables: the portfolio's outstanding principal balance and the portfolio's promised interest payments.

Consider the complete distribution over $m_t(r)$, the start-of-period balance of a loan with interest rate r, as a state variable. Banks can freely choose their end-of-period holdings of these loans $\tilde{m}_t(r)$ by trading in the secondary market at price $q^m(r)$. In this case, the bank's problem is to choose new debt issuance L_t^*, new deposits B_{t+1}^I and end-of-period loan holdings $\tilde{m}_t(r)$ to maximize shareholder value

$$V^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{m}_t(r), B_{t+1}^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right], \tag{39}$$

subject to the definition of net worth:

$$W_t^I = \underbrace{\int \left[X_t + Z_{A,t} r + Z_{M,t} \Big((1-\delta) + \delta Z_{R,t} \Big) \right] m_t(r)\, dr}_{\text{payments on existing debt}} + \underbrace{\int q_t^m(r) \delta (1 - Z_{R,t}) Z_{M,t} m_t(r)\, dr}_{\text{secondary market sales}} - \underbrace{\bar{\pi}^{-1} B_t^I}_{\text{old deposits}}, \tag{40}$$

asset portfolio:

$$J_t^I = \underbrace{(1 - q_t^m(r_t^*)) L_t^*}_{\text{net new debt}} + \underbrace{\int q_t^m(r) \tilde{m}_t(r)\, dr}_{\text{secondary market purchases}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}}, \tag{41}$$

and the leverage constraint:

$$q_t^f B_{t+1}^I \leq \phi^I \int q_t^m(r)\tilde{m}_t(r)\, dr, \tag{42}$$

with the law of motion (by vintage r):

$$m_{t+1}(r) = \bar{\pi}^{-1}\zeta_{p,t+1}\tilde{m}_t(r). \tag{43}$$

and where the recovery rate X_t is defined as in the main text. To obtain aggregation, we can split $q_t(r)$ into an interest-only strip with value q_t^M and a principal-only strip with value q_t^A, so that

$$q_t^m(r) = rq_t^A + q_t^M.$$

Substituting this definition into equations (40) – (43), and applying the identities

$$M_t^I = \int m_t(r)\, dr$$
$$A_t^I = \int rm_t(r)\, dr$$

yields the aggregated intermediary problem of Section 3.5. The same logic applies to the mortgage debt holdings of savers. Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant $Z_{R,t}$ across the r distribution), the prices q_t^A and q_t^M are independent of r. Furthermore, the effects of indexation are also independent of the vintage rate r.

A.3 Bank Aggregation and FOCs

Aggregation. The value of banks that do not default can be expressed recursively as:

$$V_{ND}^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I - \epsilon_t^I + \mathrm{E}_t\left[\Lambda_{t+1}^I \max\left\{V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}), 0\right\}\right], \tag{44}$$

subject to the bank leverage constraint (27), the definitions of J_t^I and W_t^I in (22) and (25), respectively, and the transition laws for the aggregate supply of IO and PO strips in (20) – (24). The value of defaulting banks to shareholders is zero.

The value of the newly started bank that replaces a bank liquidated by the government after defaulting, is given by:

$$V_R^I(\mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} - J_t^I + \mathrm{E}_t\left[\Lambda_{t+1}^I \max\left\{V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}), 0\right\}\right], \tag{45}$$

subject to the same set of constraints as the non-defaulting bank.

Beginning-of-period net worth W_t^I and the idiosyncratic profit shock ϵ_t^I are irrelevant for the portfolio choice of newly started banks. Inspecting equation (44), one can see that the optimization problem of non-defaulting banks is also independent of W_t^I and ϵ_t^I, since the value function is linear in those variables and they are determined before the portfolio decision. Taken together, this implies that all banks will choose identical portfolios at the end of the period. This property gives rise to aggregation, as we show next.

Starting from the value function in (44), we can define a value function net of the idiosyncratic profit shock:

$$V^I(W_t^I, \mathcal{S}_t) = V_{ND}^I(W_t^I, \mathcal{S}_t) + \epsilon_t^I,$$

such that we can equivalently write the optimization problem of the non-defaulting bank after the default decision as:

$$V^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I, 0 \right\} \right], \tag{46}$$

subject to the same set of constraints as the original problem.

We can now take the expectation with respect to ϵ_t^I of the term in the expectation operator:

$$\begin{aligned} &\mathbb{E}_\epsilon \left[\max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I, 0 \right\} \right] \\ &\quad = \mathrm{Prob}_\epsilon \left(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \mathbb{E}_\epsilon \left[V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I \,|\, \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right] \\ &\quad = F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right), \end{aligned} \tag{47}$$

with $\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \,|\, \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right]$ as in the main text. Inserting (47) into (46) gives the value function in (26) in the main text.

The value of the newly started bank with zero net worth is simply the value in (26) evaluated at $W_t^I = 0$: $V_R^I(\mathcal{S}_t) = V^I(0, \mathcal{S}_t)$.

FOCs. To derive the first-order conditions for the bank problem, we formulate the Lagrangian

$$\begin{aligned} \mathcal{L}^I(W_t^I, \mathcal{S}_t) = &\max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} \min_{\lambda_t^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right] \\ &+ \lambda_t^I \left(\phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - B_{t+1}^I \right) \right), \end{aligned} \tag{48}$$

and further conjecture that

$$V^I(W_t^I, \mathcal{S}_t) = W_t^I + \mathcal{C}(\mathcal{S}_t), \tag{49}$$

where $\mathcal{C}(\mathcal{S}_t)$ is a function of the aggregate state variables but not individual bank net worth.

Before differentiating (48) to obtain first-order conditions, note that the derivative of the term in the expectation operator with respect to future wealth, after substituting in this guess, is:

$$\frac{\partial}{\partial W_{t+1}^I} F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}) \right) \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right)$$

$$= \frac{\partial}{\partial W^I_{t+1}} \left[F^I_\epsilon \left(W^I_{t+1} + \mathcal{C}(\mathcal{S}_{t+1}) \right) \left(W^I_{t+1} + \mathcal{C}(\mathcal{S}_{t+1}) \right) - \int_{-\infty}^{W^I_{t+1} + \mathcal{C}(\mathcal{S}_{t+1})} \epsilon f^I_\epsilon(\epsilon)\, d\epsilon \right]$$
$$= F^I_\epsilon \left(W^I_{t+1} + \mathcal{C}(\mathcal{S}_{t+1}) \right) \equiv F^I_{\epsilon,t+1}.$$

Using this result, and differentiating with respect to L^*_t, $\tilde{M}^I_t$, $\tilde{A}^I_t$, B^I_{t+1}, and λ^I_t, respectively, gives the first-order conditions:

$$1 = q^M_t + r^*_t q^A_t, \tag{50}$$

$$q^M_t = \frac{\mathrm{E}_t \left\{ \Lambda^I_{t+1} F^I_{\epsilon,t+1} \bar{\pi}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{M,t+1} \Big((1-\delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) q^M_{t+1}) \Big) \right] \right\}}{(1 - \phi^I \lambda^I_t)}, \tag{51}$$

$$q^A_t = \frac{\mathrm{E}_t \left\{ \Lambda^I_{t+1} F^I_{\epsilon,t+1} \bar{\pi}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \Big(1 + \delta (1 - Z_{R,t+1}) q^A_{A,t+1} \Big) \right] \right\}}{(1 - \phi^I \lambda^I_t)}, \tag{52}$$

$$q^f_t = \mathrm{E}_t \Big[\Lambda^I_{t+1} F^I_{\epsilon,t+1} \bar{\pi}^{-1} \Big] + \lambda^I_t, \tag{53}$$

and the usual complementary slackness condition for λ^I_t.

Recalling the definition of J^I_t:

$$J^I_t = (1 - r^*_t q^A_t - q^M_t) L^*_t + q^A_t \tilde{A}^I_t + q^M_t \tilde{M}^I_t - q^f_t B^I_{t+1},$$

we note that the term in front of L^*_t is zero due to FOC (50). We can substitute out prices q^M_t, q^A_t, and q^f_t from FOCs (51)-(53), both in J^I_t and in the constraint term in (48). Further inserting our guess from (49) on the left-hand side of (48), and canceling and collecting terms, we get:

$$\mathcal{C}(\mathcal{S}_t) = \mathrm{E}_t \left[\Lambda^I_{t+1} F^I_\epsilon \left(W^I_{t+1} + \mathcal{C}(\mathcal{S}_{t+1}) \right) \left(\mathcal{C}(\mathcal{S}_{t+1}) - \epsilon^{I,-}_{t+1} \right) \right], \tag{54}$$

which confirms the conjecture. $\mathcal{C}(\mathcal{S}_t)$ is the recursively defined value of the bankruptcy option to the bank. Note that without the option to default,

$$\epsilon^{I,-}_{t+1} = \mathrm{E}_\epsilon \left[\epsilon^I_{t+1} \right] = 0.$$

Then the equation in (54) implies that $\mathcal{C}(\mathcal{S}_t) = 0$ and thus $V^I(W^I_t, \mathcal{S}_t) = W^I_t$. However, if the bank has the option to default, its value generally exceeds its financial wealth W^I_t by the bankruptcy option value $\mathcal{C}(\mathcal{S}_t)$. Deposit insurance creates bank franchise value.

A.4 Borrower Optimality

The optimality condition for new mortgage debt,

$$1 = \Omega_{M,t} + r^*_t \Omega_{A,t} + \lambda^{LTV}_t, \tag{55}$$

equalizes the benefit of taking on additional debt – \$1 today – to the cost of carrying more debt in the future, both in terms of carrying more principal ($\Omega_{M,t}$) and higher interest payments ($\Omega_{A,t}$), plus the shadow cost of tightening the LTV constraint. The marginal continuation costs are defined recursively:

$$\Omega_{M,t} = \mathbb{E}_t \left\{ \Lambda^B_{t+1} \bar{\pi}^{-1} \zeta_{p,t+1} Z_{M,t+1} \Big[(1-\delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) \Omega_{M,t+1} \Big] \right\}, \tag{56}$$

$$\Omega_{A,t} = \mathbb{E}_t \left\{ \Lambda^B_{t+1} \bar{\pi}^{-1} \zeta_{p,t+1} Z_{A,t+1} \Big[(1-\tau) + \delta(1 - Z_{R,t+1}) \Omega_{A,t+1} \Big] \right\}, \tag{57}$$

where an extra unit of principal requires a regular principal amortization payment of $(1-\delta)$ in the case of non-default, plus payment of the face value of prepaid debt, plus the continuation cost of non-prepaid debt. An extra promised payment requires a tax-deductible payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

The optimality condition for housing services consumption sets the rental rate equal to the marginal rate of substitution between housing services and nondurables:

$$\rho_t = \frac{u_{H,t}}{u_{C,t}} = \left(\frac{\xi_t}{1-\xi_t} \right) \left(\frac{C^B_t}{H^B_t} \right)$$

The borrower's optimality condition for new housing capital is:

$$p_t = \frac{\mathbb{E}_t \Big\{ \Lambda^B_{t+1} \Big[\rho_{t+1} + Z_{K,t+1} p_{t+1} \Big(1 - \nu^K - (1 - Z_{R,t+1}) \lambda^{LTV}_{t+1} \phi^K \Big) \Big] \Big\}}{1 - \lambda^{LTV}_t \phi^K}. \tag{58}$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by $(1 - Z_{R,t+1}) \lambda^{LTV}_{t+1} \phi^K$ because if the borrower does not choose to refinance, which occurs with probability $1 - Z_{R,t+1}$, then she does not use the unit of housing to collateralize a new loan, and therefore does not receive the collateral benefit.

The optimal refinancing rate is:

$$\begin{aligned} Z_{R,t} = \Gamma_\kappa \Bigg\{ & \underbrace{(1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t}) \left(1 - \frac{\delta Z_{M,t} M_t}{Z_{N,t} M^*_t} \right)}_{\text{equity extraction incentive}} + \underbrace{\Omega_{A,t} \left(\bar{r}_t - r^*_t \right)}_{\text{interest rate incentive}} \\ & - \underbrace{p_t \lambda^{LTV}_t \phi^K \left(\frac{Z_{N,t} K^*_t - Z_{K,t} K^B_t}{Z_{N,t} M^*_t} \right)}_{\text{collateral expense}} \Bigg\} \end{aligned} \tag{59}$$

where $\bar{r}_t = A^B_t / M^B_t$ is the average interest rate on existing debt. The "equity extraction incentive" term represents the net gain from obtaining additional debt at the *existing* interest rate, while "interest rate incentive" term represents the gain from moving from the existing to new interest rate. The stronger these incentives, the higher the refinancing rate. The "collateral expense" term arises because housing trades at a premium relative to the present value of its housing service flow due to its collateral value. If the borrower intends to obtain new debt by buying more housing collateral, the cost of paying this premium must be taken into account.

The optimality condition for the default rate pins down the default threshold $\bar{\omega}_t^U$ as a function of the aggregate state, as well as the value of the local component ($\omega_{i,t}^L$):

$$\bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{\iota_\omega} \left(Q_{A,t} A_t^B + Q_{M,t} M_t^B \right)}{\omega_{i,t}^L Q_{K,t} K_t^B}, \tag{60}$$

where $Q_{A,t}$ and $Q_{M,t}$ are the marginal benefits of discharging interest payments and principal, respectively, and $Q_{K,t}$ is the marginal continuation value of housing, defined by:

$$Q_{A,t} = \underbrace{(1-\tau)}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{A,t}}_{\text{continuation cost}} \tag{61}$$

$$Q_{M,t} = \underbrace{(\delta Z_{R,t} + (1-\delta))}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{M,t}}_{\text{continuation cost}} \tag{62}$$

$$Q_{K,t} = \Big[\underbrace{Z_{R,t}}_{\text{refi case}} + \underbrace{(1 - Z_{R,t}) \left(1 - \lambda_t^{LTV} \phi^K \right)}_{\text{no refi case}} - \underbrace{\nu^K}_{\text{maint.}} \Big] p_t. \tag{63}$$

The marginal value of housing $Q_{K,t}$ is equal to the full market price p_t net of maintenance if used to collateralize a new loan (i.e., if the borrower refinances), but is worth less if the borrower does not refinance next period due to the loss of collateral services. Equation (60) relates the benefit of defaulting on debt, which is eliminating both the current payment and continuation cost, potentially indexed by $\omega_{i,t}^L$, against the cost of losing a marginal unit of housing, which is scaled by both $\omega_{i,t}^L$ and $\omega_{i,t}^U$. Default occurs when the market value of the debt exceeds the market value of the collateral, i.e., the mark-to-market LTV exceeds 1. The market value of debt reflects the option value of default and prepayment. Because the option to delay default is valuable to the borrower, the market value of the debt tends to be below the book value of the debt. In other words, it can be optimal to continue servicing the debt when the book LTV (which contains the book value of debt in the numerator and ignores the value of delay) exceeds 1. In the case of local indexation ($\iota_\omega = 1$), the market LTV is immunized from shocks to local house prices.

A.5 Intermediary Optimality

Bank owner. Given their preferences (2), the bank owner's budget constraint in (19) always holds with equality and the household's only choice, consumption C_t^I, is determined from the budget constraint. Bank owners trade equity shares of banks and REO firms in competitive markets, and one could derive the market value of these firms from the intermediary household's first-order conditions. In equilibrium, the representative bank owner holds 100% of the outstanding shares, and thus these optimality conditions are not needed to solve for the model's dynamics. Nonetheless, the bank owner's optimization problem gives rise to the stochastic discount factor Λ^I_{t+1}.

Banks. Optimality conditions for banks are discussed in Appendix A.3.

REO firms. The optimality condition for REO housing is:

$$p^{REO}_t = \mathbb{E}_t \left\{ \Lambda^I_{t+1} \Big[\rho_{t+1} - \nu^{REO} p_{t+1} + S^{REO} p_{t+1} + (1 - S^{REO}) p^{REO}_{t+1} \Big] \right\}. \tag{64}$$

The right-hand side is the present discounted value of holding a unit of REO housing next period. This term is in turn made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers, and the portion kept in the REO state.

A.6 Saver Optimality

The savers' optimality condition for deposits, which are nominal contracts, is:

$$q^f_t = \mathbb{E}_t \Big[\Lambda^S_{t+1} \bar{\pi}^{-1} \Big]. \tag{65}$$

Savers also trade IO and PO strips in the secondary market. Their choice variable is the amount of PO strips $\tilde{M}^S_t$. To write the first-order condition for this choice variable, it is useful to define

$$\hat{r}_t = \frac{\hat{A}^I_t + \hat{A}^S_t}{\hat{M}^I_t + \hat{M}^S_t},$$

which is the effective interest rate paid on all debt supplied in secondary markets (including new debt).

Since savers always hold IO and PO strips in the same proportion as the market supply, their choice of $\tilde{M}^S_t$ implies a choice of IO strips of $\tilde{A}^S_t = \hat{r}_t \tilde{M}^S_t$. The FOC for $\tilde{M}^S_t$ is therefore

$$\begin{aligned} q^M_t + \hat{r}_t q^A_t + \varphi_0 (\tilde{M}^S_t)^{\varphi_1 - 1} \\ = \mathbb{E}_t \left\{ \Lambda^S_{t+1} \bar{\pi}^{-1} \zeta_{p,t+1} \left[X_{t+1} + Z_{M,t+1} \Big((1-\delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) q^M_{t+1} \Big) \right] \right\} \\ + \hat{r}_t \mathbb{E}_t \left\{ \Lambda^S_{t+1} \bar{\pi}^{-1} \zeta_{p,t+1} \left[Z_{A,t+1} \Big(1 + \delta (1 - Z_{R,t+1}) q^A_{A,t+1} \Big) \right] \right\} + \lambda^S_t, \end{aligned} \tag{66}$$

where λ_t^S is the Lagrange multiplier on the no-shorting constraint

$$\tilde{M}_t^S \geq 0. \tag{67}$$

The marginal cost of buying the combined portfolio of IO and PO strips on the left-hand side consists of the security prices, and the marginal portfolio holding cost $\varphi_0(\tilde{M}_t^S)^{\varphi_1 - 1}$. Savers have a comparative disadvantage (relative to banks) at holding mortgage securities governed by the magnitude of the cost.

B Online Appendix: Model Solution

B.1 Model Equations and State Variables

The model's equilibrium can be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm, Kryczka, and Kubler (2018) analyze theoretical existence properties in this class of models and discuss the literature.

The endogenous aggregate state of the model can be represented in several ways. First, the state variables need to determine the wealth distribution among the types of optimizing agents: borrowers (W^B), banks and bank owners (W^I), and savers. Secondly, since our model features long-term mortgages with refinancing costs, we need to track the total amount of outstanding mortgage debt (M^B) and interest owed (A^B). Finally, since foreclosed houses incur higher maintenance than regular houses, we need to keep track of the stock of REO properties (K^{REO}). For the extended model with government tax smoothing, the outstanding stock of government debt (B^G) becomes yet another state variable.

The minimal set of aggregate state variables is thus $[A_t^B, M_t^B, K_t^{REO}, W_t^I, B_t^G]$, and the complete vector of aggregate state variables is $\mathcal{S}_t = [Y_t, \sigma_{\omega,t}, A_t^B, M_t^B, K_t^{REO}, W_t^I, B_t^G]$. Using $[A_t^B, M_t^B, K_t^{REO}]$, we can directly compute beginning-of-period borrower wealth. Combining the budget constraints of all agents, we can back out saver wealth knowing borrower wealth, intermediary wealth, and government debt.

Functions. We can characterize the equilibrium as a system of 13 functional equations that we list below. The equilibrium objects to be computed, such as agent's optimal choices and market-clearing prices, are functions of the model's aggregate state variables and represent a solution to the functional equations. The functions to be computed are for borrowers:

F1. consumption $c^B(\mathcal{S}_t)$,

F2. the refinancing rate $Z^R(\mathcal{S}_t)$,

F3. the Lagrange multiplier on the LTV constraint for refinancers $\lambda^{LTV}(\mathcal{S}_t)$,

for bank owners and banks:

F4. consumption $c^I(\mathcal{S}_t)$,

F5. the Lagrange multiplier on the bank leverage constraint $\lambda^I(\mathcal{S}_t)$,

for savers:

F6. consumption $c^S(\mathcal{S}_t)$,

F7. direct holdings of mortgage debt $\tilde{M}^S(\mathcal{S}_t)$,

F8. the Lagrange multiplier on the no-shorting constraint for mortgage debt $\lambda^S(\mathcal{S}_t)$,

and market prices for:

F9. PO strips $q^M(\mathcal{S}_t)$,

F10. IO strips $q^A(\mathcal{S}_t)$,

F11. housing capital $p(\mathcal{S}_t)$,

F12. REO housing capital $p^{REO}(\mathcal{S}_t)$, and

F13. riskfree bonds $q^f(\mathcal{S}_t)$.

All other choice variables and model outcomes have explicit closed-form solutions given the state variables and these 13 functions.

Equations. In equilibrium, functions F1 – F13 must jointly satisfy equations (E1) – (E13) listed below at each point in the aggregate state space. The equations are intertemporal first-order conditions to the agents' optimization problems, complementary slackness conditions for constraints, and one market clearing condition. For borrowers, we have the FOC for new borrowing, new housing capital, and the refinancing rate, and the complementary slackness condition for the LTV constraint (E1 – E4). These equilibrium conditions are derived in Section A.4 and correspond to equations (55), (58), (59), and (18). For the intermediary sector, we have banks' FOC for IO strips, PO strips, and deposits, plus the complementary slackness condition on banks' regulatory constraint (E5 – E8), which are derived in A.3 and correspond to equations (51) – (53) and (27). The REO firms' contribute a FOC for purchases of REO properties (E9) in equation (64). For savers, we have the FOC for riskfree debt and mortgage holdings, and the complementary slackness condition for the no-shorting constraint on mortgages (E10 – E12), which are derived in Section A.6 and correspond to equations (65), (66), and (67). The final equation is the market clearing condition for riskfree debt (E13).

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV} \tag{E1}$$

$$p_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^B \left[\rho_{t+1} + Z_{K,t+1} p_{t+1} \left(1 - \nu^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K \right) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K} \tag{E2}$$

$$\begin{aligned} Z_{R,t} = \Gamma_\kappa \Bigg\{ & (1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t}) \left(1 - \frac{\delta Z_{M,t} M_t}{Z_{N,t} M_t^*} \right) + \Omega_{A,t} (\bar{r}_t - r_t^*) \\ & - p_t \lambda_t^{LTV} \phi^K \left(\frac{Z_{N,t} K_t^* - Z_{K,t} K_t^B}{Z_{N,t} M_t^*} \right) \Bigg\} \end{aligned} \tag{E3}$$

$$0 = \lambda_t^{LTV} (\phi^K p_t K_t^* - M_t^*) \tag{E4}$$

$$q_t^M = \frac{\mathbb{E}_t\left\{\Lambda^I_{t+1}F^I_{\epsilon,t+1}\bar{\pi}^{-1}\zeta_{p,t+1}\left[X_{t+1}+Z_{M,t+1}\Big((1-\delta)+\delta Z_{R,t+1}+\delta(1-Z_{R,t+1})q^M_{t+1})\Big)\right]\right\}}{(1-\phi^I\lambda^I_t)} \tag{E5}$$

$$q_t^A = \frac{\mathbb{E}_t\left\{\Lambda^I_{t+1}F^I_{\epsilon,t+1}\bar{\pi}^{-1}\zeta_{p,t+1}\left[Z_{A,t+1}\Big(1+\delta(1-Z_{R,t+1})q^A_{t+1}\Big)\right]\right\}}{(1-\phi^I\lambda^I_t)} \tag{E6}$$

$$q_t^f = \mathbb{E}_t\Big[\Lambda^I_{t+1}F^I_{\epsilon,t+1}\bar{\pi}^{-1}\Big]+\lambda^I_t \tag{E7}$$

$$0 = \lambda^I_t\left(\phi^I\left(q^A_t\tilde{A}^I_t+q^M_t\tilde{M}^I_t\right)-B^I_{t+1}\right) \tag{E8}$$

$$p_t^{REO} = \mathbb{E}_t\left\{\Lambda^I_{t+1}\Big[\rho_{t+1}-\nu^{REO}p_{t+1}+S^{REO}p_{t+1}+(1-S^{REO})p^{REO}_{t+1}\Big]\right\} \tag{E9}$$

$$q_t^f = \mathbb{E}_t\Big[\Lambda^S_{t+1}\bar{\pi}^{-1}\Big] \tag{E10}$$

$$\begin{aligned} q^M_t + \hat{r}_t q^A_t + \varphi_0(\tilde{M}^S_t)^{\varphi_1-1} \\ &= \mathbb{E}_t\left\{\Lambda^S_{t+1}\bar{\pi}^{-1}\zeta_{p,t+1}\left[X_{t+1}+Z_{M,t+1}\Big((1-\delta)+\delta Z_{R,t+1}+\delta(1-Z_{R,t+1})q^M_{t+1})\Big)\right]\right\} \\ &\quad + \hat{r}_t\mathbb{E}_t\left\{\Lambda^S_{t+1}\bar{\pi}^{-1}\zeta_{p,t+1}\left[Z_{A,t+1}\Big(1+\delta(1-Z_{R,t+1})q^A_{t+1}\Big)\right]\right\} \end{aligned} \tag{E11}$$

$$0 = \lambda^S_t\tilde{M}^S_t \tag{E12}$$

$$0 = B^I_{t+1}+B^G_{t+1}-B^S_{t+1}. \tag{E13}$$

With the exception of (E13), these equations represent the minimal set of conditions that define the economy's equilibrium. In other words, functions F1 – F13 are only implicitly defined by the system (E1) – (E13), and we need to solve for equilibrium by numerically finding the root of the system of equations. The market clearing condition (E13) is a linear relationship, and we could use this condition to explicitly solve for the consumption of bank owners (c^I) or savers (c^S), thus reducing the size of the system to 12 equations in 12 unknowns. However, it is useful for numerical stability to retain all consumption functions as solution variables, since it allows to enforce strict positivity of these functions when searching for the solution.

More generally, conditions (E1) – (E13) rely on various other equilibrium relationships that we use to compute expressions used in the equations, given the state variables $\mathcal{S}_t$ and functions F1 – F13. For example, borrower continuation values $\Omega_{M,t}$ and $\Omega_{A,t}$ are defined in equations (56) and (57). Similarly, the survival rates for debt and housing capital $(Z_{N,t}, Z_{K,t}, Z_{M,t}, Z_{A,t})$ appearing in (E2), (E3), (E5), (E6) and (E11) are defined in equations (8) – (10), and use optimal default threshold (60). The stochastic discount factors Λ^j_{t+1}, for $j = B, I, S$, are defined in (38) and require us to track recursive utility U^j_t as in (2) for each type of household. We further use budget constraints and market clearing conditions to calculate borrowers' and banks' portfolio choices, given their consumption and savers' holdings of mortgages, which are all contained in F1 – F13.

Transitions. The rational expectations equilibrium requires that agents correctly forecast the law of motion of the aggregate state variables. The exogenous state variables $[Y_t, \sigma_{\omega,t}]$ follow a discrete Markov chain, with states and transition probabilities known to agents. The endogenous state variables evolve according to equations (T1) – (T5):

$$M^B_{t+1} = \bar{\pi}^{-1}\zeta_{p,t+1}\Big[Z_{R,t}Z_{N,t}M^*_t + \delta(1-Z_{R,t})Z_{M,t}M^B_t\Big] \tag{T1}$$

$$A^B_{t+1} = \bar{\pi}^{-1}\zeta_{p,t+1}\Big[Z_{R,t}Z_{N,t}r^*_t M^*_t + \delta(1-Z_{R,t})Z_{A,t}A^B_t\Big] \tag{T2}$$

$$K^{REO}_{t+1} = (1-S^{REO})K^{REO}_t + I^{REO}_t \tag{T3}$$

$$\begin{aligned} W^I_{t+1} &= X_{t+1}M^I_{t+1} + Z_{M,t+1}\Big(1-\delta+\delta Z_{R,t+1}\Big)M^I_{t+1} + Z_{A,t+1}A^I_{t+1} \\ &\quad + \delta(1-Z_{R,t+1})\left(q^A_{t+1}Z_{A,t+1}A^I_{t+1} + q^M_{t+1}Z_{M,t+1}M^I_{t+1}\right) - \bar{\pi}^{-1}B^I_{t+1} \end{aligned} \tag{T4}$$

$$B^G_{t+1} = \frac{1}{q^f_t}\left(\bar{\pi}^{-1}B^G_t + \text{bailout}_t - T_t\right). \tag{T5}$$

Equation (T1) and (T2) directly correspond to the laws of motion of borrower debt in (11) – (12). (T3) is the law of motion of REO capital in (31). (T4) is the definition of bank net worth next period as function of today's portfolio, (25), where we have substituted for CF_{t+1} and EDV_{t+1} from (15) and (16). Finally, (T5) is the government budget constraint (35).

B.2 Numerical Solution Method

We solve the model numerically using policy function iteration (Judd, 1998). Our global, nonlinear method allows us to compute a numerical solution to the economy's equilibrium with high accuracy. While a local method that approximates the equilibrium around the deterministic "steady-state" would be simpler, it would not provide a reliable approximation to our model economy. First, portfolio restrictions such as banks' leverage constraints are only occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Further, local methods have difficulties in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs, as is the case for the quantitative model in this paper. Finally, in models with rarely occurring bad shocks (such as the financial recessions in our model), the steady state used by local methods may not properly capture the ergodic distribution of the true dynamic equilibrium due to precautionary motives and risk premia.

Global projection methods avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space.

B.3 Solution Procedure

The projection-based solution approach used in this paper has three main steps.

Step 1. **Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our solution method requires approximation of three sets of functions defined on the domain of the state variables. The first set, the "policy" functions, determine the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the functions F1 – F13 listed in Section B.1. The second set, the "transition" functions, determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks, corresponding to transition laws (T1) – (T5) in Section B.1. The third set are "forecasting functions". They map the state into variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions, but contain some additional information, for example the recursive utility of each agent.

Step 2. **Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate. The sub-steps are:

A. **Initialize** the algorithm by specifying a guess for the policy and transition functions.

B. **Compute forecasting values.** For each point in the discretized state space, perform the steps:

   i. Evaluate the transition functions at each possible realization of the aggregate state combined with each possible realization of the exogenous shocks.

   ii. Evaluate the forecasting functions at these future state variable realizations.

   The end result is a matrix, with each entry being a vector of the next-period realization of the forecasting functions for each possible combination of current state and next-period exogenous state.

C. **Solve system of nonlinear equations.** At each point in the discretized state space, solve the system of nonlinear equations that characterize equilibrium in the equally many "policy" variables, given the forecasting matrix from step B. This amounts to solving a nonlinear system of 13 equations in 13 unknowns at each of the roughly 30,000 points in the state space, with the unknowns being the functions values for F1 – F13 and the equations given by (E1) – (E13).

Expectations are computed as weighted sums, with the weights being the conditional transition probabilities of the exogenous states. The expressions in expectations generally depend on the forecasting matrix, which we pre-computed in step B.

To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions from the last iteration as initial guess. More on these issues in subsection B.4 below.

The final output of this step is a matrix, where each row is the solution vector that solves the system (E1) – (E13) at a specific point in the discretized state space. This is the numerical representations of functions F1 – F13.

D. **Update forecasting, transition and policy functions.** Given the policy matrix from step C, update the policy and forecasting functions.

   Finally, updating transition functions for the endogenous state variables according to (T1) – (T5) gives the complete set of functions for the next iteration.

E. **Check convergence.** Compute a distance measure on the forecasting, policy and/or transition function between current and previous iterate. If the distance is below the convergence threshold, stop and use the current functions as approximate solution. Otherwise reset all functions to the current iterate and go to step B.

Step 3. **Simulate the model for many periods using approximated functions.** Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

B.4 Implementation

Solving the system of equations. We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB's fsolve). This nonlinear equation solver uses a variant of Newton's method to find a "zero" of the system. We employ several simple modifications of the system (E1) – (E13) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with a constraint. Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, given the utility function, optimal consumption is always strictly positive. To avoid that the solver tries out negative consumption values (and thus output becomes ill-defined), we use $\log(c_t^j)$ as solution variable for the solver. This means the solver can make consumption arbitrarily small, but not negative.

Grid configuration. For the benchmark case, the grid points in each state dimension are as follows

- Y: We discretize Y into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points $\{Y\}_{j=1}^{5}$ the 5×5 Markov transition matrix Π_Y between them to match the volatility and persistence of GDP growth. This yields the possible realizations for Y: $[0.9834, 0.9913, 0.9993, 1.00731.0154]$.
- σ_ω: $[0.2, 0.25]$ (see calibration)
- A^B: $[0.0330, 0.0350, 0.0370, 0.0390, 0.0410, 0.0430]$
- M^B: $[2.2000, 2.3083, 2.4167, 2.5250, 2.6333, 2.7417, 2.8500]$
- K^{REO}: $[0, 0.0100, 0.0200, 0.0300, 0.0400, 0.0500]$
- W^I: $[0.0100, 0.0300, 0.0500, 0.0600, 0.0800, 0.1100, 0.1300, 0.1500, 0.1900, 0.2100, 0.2300]$

The total state space grid has 27,720 points. The grid boundaries, placement and number of points have to be readjusted for each experiment, since the ergodic distribution of the state variables depends on parameters. For example, in the model with government tax smoothing, government debt B^G becomes an additional aggregate state variable, as its value is generally greater than 0. Thus, we need to add a grid for B^G. Finding the right values for the boundaries is a matter of experimentation.

Generating an initial guess and iteration scheme. To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic "steady-state" of the model under the assumption that the bank leverage constraint is binding and housing risk is low. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, this steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these "failed" points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually finds solutions at all points.

To determine convergence, we check absolute errors in the value functions of households. Out of all functions we approximate during the solution procedure, these exhibit the slowest convergence.

We stop the solution algorithm when the mean absolute difference between two iterations, and for all points in the state space, falls below 1e-4. We stop the procedure after 150 iterations.

In some cases, our grid boundaries are wider than necessary, in the sense that the simulated economy never visits the areas near the boundary on its equilibrium path. Local convergence in those areas is usually very slow, but not relevant for the equilibrium path of the economy. If the algorithm has not achieved convergence after 150 iterations, we nonetheless stop the procedure and simulate the economy. If the resulting simulation produces low relative errors (see step 3 of the solution procedure), we accept the solution. After the 150 iterations, our simulated model economies either achieve acceptable accuracy in relative errors, or if not, the cause is a badly configured state grid. In the latter case, we need to improve the grid and restart the solution procedure. Additional iterations beyond 150 do not change any statistics of the simulated equilibrium path for any of the simulations we report.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 16 CPU cores of a single HPC node. From computing the initial guess and analytic Jacobian to simulating the solved model, the total running time for the benchmark calibration is about 2 hours.

Simulation. To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 1,000 periods. The starting point of the simulation is the ergodic mean of the state variables. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity ($Y = .9993$) and normal housing risk ($\sigma_\omega = 0.2$). The "impulse" in the second period is either only a bad endowment shock, or both low endowment and a housing risk shock ($\sigma_\omega = 0.25$). For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition.

Evaluating the solution. To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (E1) – (E13) and the transition functions (T1) – (T5) along the simulated path. For equations involving expectations (such as (E2)), this requires evaluating the transition and forecasting function as in step 2B at the current state. For each equation,

Table B.1: Computational Errors for Benchmark

Equation			**Percentile**		
	50th	75th	95th	99th	Max
E1	0.000482	0.000517	0.000575	0.000599	0.000614
E2	0.002198	0.002229	0.002279	0.002301	0.002316
E3	0.000101	0.000131	0.00016	0.000168	0.000239
E4	0.00212	0.002997	0.007354	0.015504	0.021663
E5	0.002447	0.003344	0.007591	0.015876	0.021977
E6	0.000899	0.001189	0.001652	0.001775	0.001836
E7	0.002564	0.003556	0.008165	0.017052	0.023623
E8	0.000188	0.000211	0.00023	0.000332	0.00047
E9	9.18E-06	1.53E-05	6.4E-05	0.00014	0.000178
E10	0.00236	0.003381	0.004594	0.005102	0.007374
E11	0.000186	0.000247	0.000321	0.000451	0.001205
E12	0.000304	0.000452	0.002106	0.008908	0.012928
E13	8.58E-05	0.000116	0.000159	0.000211	0.000262

we divide both sides by a sensibly chosen endogenous quantity to yield "relative" errors to make the scale of the errors economically meaningful and comparable across equations. In practice, this means that we divide both sides of each equation to normalize either the RHS or the LHS to 1.

Table B.1 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 100^{th} percentiles during the 5,000 period simulation of the model. Median errors are very small for all equations, with even maximum errors only causing small approximation mistakes. Errors are comparably small for most experiments we report.

These errors are small by construction when calculated at the points of the discretized state grid, since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the equilibrium functions are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

C Online Appendix: Model Extensions

C.1 IO/PO Indexation

The default thresholds with interest-only (IO) and principal-only (PO) mortgage payment indexation are given by:

$$\text{Interest Only}: \qquad \bar{\omega}_t^U = \frac{Q_{A,t}A_t + (\omega_{i,t}^L)^{\iota_\omega} Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t} K_t^B}$$

$$\text{Principal Only}: \qquad \bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{\iota_\omega} Q_{A,t}A_t + Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t} K_t^B}$$

which are identical to (60) with the exception that only one component or the other in the numerator is indexed, but not both. Additionally, $Z_{M,t}$ is replaced by $Z_{N,t}$ and $\zeta_{p,t}$ is replaced by 1 when scaling principal balances in the interest-only case. Symmetrically, $Z_{A,t}$ is replaced by $Z_{N,t}$ and $\zeta_{p,t}$ is replaced by 1 when scaling interest payments A_t in the principal-only case.

C.2 Persistent ω Processes

This section provides details on how our assumption of i.i.d. ω shocks map to a more realistic set of persistent processes under symmetric indexation, and how we adjust our formulas to directly make use of the persistent process specification for the asymmetric indexation case in which they are no longer equivalent.

Symmetric Indexation. First, we demonstrate the link between our i.i.d. assumption and persistent processes under symmetric indexation. We begin by noting that under symmetric indexation, only the change in the level of ω^L between the origination of the loan and the current period is a sufficient statistic for the total indexation. Define $LogTotalIndexation_{s,t}$ to be the total share of the loan that is either forgiven or added to the loan due to indexation from origination at time s to the current period at time t, in logs. This quantity is equal to

$$LogTotalIndexation_{s,t} = \sum_{k=s}^{t} \iota_\omega \Delta \log \omega_k^L = \iota_\omega (\log \omega_t^L - \log \omega_s^L). \tag{68}$$

Normalizing $\log \omega_s^L = 0$ at origination, which is the appropriate normalization to ensure that loans in all areas begin with the same leverage, we find that log total indexation is equal to $\iota_\omega \log \omega_t^L$. Thus, only the distribution of $\log \omega_t^L$ matters for determining total indexation. In principle, this distribution should vary with the length of time since origination, which would require tracking each vintage of loans separately. To keep our model tractable, we assume that $\log \omega_t^L$ is drawn from its unconditional distribution Γ_ω^L, which becomes exactly correct as the rate at which loans are renewed (Z_R) goes to zero. Assuming a continuum of infinitesimal locations, this implies that the cross-sectional dispersion of the ω distribution is equal to that of its stationary distribution. Since we have complete risk sharing within the borrower family, the individuals of the members are otherwise irrelevant beyond their draws of ω, so it is without loss of generality to draw i.i.d. realizations from this distribution.

For our specific parametric form, recall that we model the ω distribution as being drawn log-normal with mean in levels equal to zero and variance in logs equal to σ_t^2. To micro-found this particular distribution, we can assume that each underlying process for ω follows an AR(1) in logs. Specifically, we define two components following:

$$\log \tilde{\omega}_{i,t}^L = \rho_\omega \log \tilde{\omega}_{i,t-1}^L + \sqrt{1-\rho_\omega^2} e_{i,t}^L, \qquad e_{i,t}^L \sim N(0,\alpha) \tag{69}$$

$$\log \tilde{\omega}_{i,t}^U = \rho_\omega \log \tilde{\omega}_{i,t-1}^U + \sqrt{1-\rho_\omega^2} e_{i,t}^U, \qquad e_{i,t}^U \sim N(0,1-\alpha) \tag{70}$$

These unscaled processes have log-normal distributions as their stationary distributions, although the means in levels are nonzero, and the variances in logs are α and $1-\alpha$ respectively. To obtain our final components with the correct means and time-varying variances, we define

$$\log \omega_{i,t}^L = \mu_{L,t} + \sigma_{L,t} \log \tilde{\omega}_{i,t} \tag{71}$$

$$\log \omega_{i,t}^U = \mu_{U,t} + \sigma_{U,t} \log \tilde{\omega}_{i,t} \tag{72}$$

where the μ terms are chosen to ensure a mean of one in levels.

Asymmetric Indexation. Under asymmetric indexation, the equivalence between our i.i.d. draws from the stationary distribution and the persistent processes no longer holds. With symmetric indexation, only the total change in ω up to time t matters, and not the specific path of changes along the way, because upward and downward changes cancel out. Under asymmetric indexation, this is no longer the case since downward movements in ω are indexed but upward movements are not. More formally, (68) no longer holds, because each $\iota_\omega \Delta \log \omega_t^L$ term should be multiplied by an indicator for whether this period's growth was positive or negative, so the sum no longer collapses.

To address this, we abandon our i.i.d. simplification for the asymmetric case, and compute indexation directly from (69) and (70). In principle, this requires us to expand our state space, since the growth rate depends on both the distribution of ω_t last period, which would require us to track the lagged crisis state as well as the current one. To avoid adding this computational burden, we approximate the growth rate distribution by assuming that the previous period's ω^L distribution is drawn from today's unconditional distribution, which is correct except in periods in which the crisis state changes, which occurs infrequently. If this ignored variation were included in our model, it would lead to more loan forgiveness, making the results under asymmetric indexation even more extreme.

To implement this scheme, we update our various model formulas, using quadrature to evaluate integrals that we can no longer compute in closed form. To begin, we update (6) and (7) to directly index to growth rates instead of levels, yielding

$$\zeta_{p,t} = \min\left\{\left(\frac{p_t}{p_{t-1}}\right)^{\iota_p}, \bar{\zeta}_p\right\}$$

$$\zeta_{\omega,t}(\omega^L_{i,t-1}, \omega^L_{i,t}) = \min\left\{\left(\frac{\omega^L_{i,t}}{\omega^L_{i,t-1}}\right)^{\iota_\omega}, \bar{\zeta}_\omega\right\}$$

As a result, the default threshold is now defined by

$$\bar{\omega}^U_t = \frac{\min\left\{\omega^L_{i,t}, \bar{\zeta}_\omega \omega^L_{i,t-1}\right\}^{\iota_\omega} (Q_{A,t}A_t + Q_{M,t}M_t)}{\omega^L_{i,t} Q_{K,t} K^B_t}$$

which, all else equal, weakly lowers the default threshold since local indexation terms become more generous when local house price growth is high. For our asymmetric indexation experiments, we set $\bar{\zeta}_p = \bar{\zeta}_\omega = 1$, implying that mortgages are never indexed upward, but only downward.

For the quantity of housing retained by the borrower, the only update needed is an extra integral to account for the dependence of $\bar{\omega}^U_t$ on both the lagged value of the local component and its innovation:

$$Z_{K,t} = \int\int\left(\int_{\omega^U_{i,t} > \bar{\omega}^U_t} \omega^U_{i,t}\, d\Gamma^U_{\omega,t}\right) \omega^L_{i,t}\, d\Gamma^L_{e,t}\, d\Gamma^L_{\omega,t-1}.$$

Finally, the quantity of debt retained by the borrowers needs to be updated both for this change in the dependence of $\bar{\omega}^U_t$, as well as the cap on how much debt can be upwardly indexed:

$$Z_{M,t} = Z_{A,t} = \int\int \underbrace{\left(1 - \Gamma^U_{\omega,t}\left(\bar{\omega}^U_t(\omega^L_{i,t-1}, e^L_{i,t})\right)\right)}_{\text{remove defaulters}} \underbrace{(\omega^L_{i,t})^{\iota_\omega}}_{\text{indexation}}\, d\Gamma^L_{e,t}\, d\Gamma^L_{\omega,t-1}$$

In the case of interest-only asymmetric indexation, $Z_{K,t}$, $Z_{A,t}$, and $Z_{N,t}$ are computed as above, $Z_{M,t} = Z_{N,t}$, and $\zeta_{p,t}$ is replaced by 1 in the transition equation for principal balances M^B_t.

The asymmetric IO-indexation case features the default threshold

$$\bar{\omega}^U_t = \frac{\min\left\{\omega^L_{i,t}, \bar{\zeta}_\omega \omega^L_{i,t-1}\right\}^{\iota_\omega} Q_{A,t}A_t + Q_{M,t}M_t}{\omega^L_{i,t} Q_{K,t} K^B_t},$$

and is otherwise symmetric, with the exception that only Z_A, and not Z_M, is adjusted for indexation.

C.3 Liquidity Defaults

This section considers a model extension where defaults are driven by both liquidity concerns (the need to stop making mortgage payments) and strategic motives.

Model. To allow for liquidity defaults, assume that fraction θ of borrowers are hit by liquidity shocks in each period. After being hit with the shock, borrowers decide whether to default on mortgage and enter foreclosure. We assume that borrowers hit with the liquidity shock only default if they their home equity is sufficiently low. Borrowers who do not default simply bear the consequences of the liquidity shock. Since we assume perfect consumption risk sharing among borrowers, the non-default case following a liquidity shock is inconsequential in our framework.

Define the $\omega_{i,t}^{U}$ threshold for default conditional on receiving a liquidity shock as $\bar{\omega}_{i,t}^{U,Liq}$. In the simplest version of our model with symmetric indexation of principal and interest, the threshold is

$$\bar{\omega}_{i,t}^{U,Liq} = \frac{(\omega_{i,t}^{L})^{\iota_\omega}\left(M_t^B + A_t^B\right)}{\Xi\omega_{i,t}^{L}(1-\nu^K)p_t K_t}, \tag{73}$$

where Ξ is a parameter that regulates the severity of the liquidity shocks. If $\Xi = 1$ and there is no indexation ($\iota_\omega = 0$), households receiving liquidity shocks default if they are under water, in the sense that the debt they owe on their house, computed as the simple sum of principal and this period's interest, $M_t^B + A_t^B$, is greater than the depreciated market value of their house, $\omega_{i,t}^{L}(1-\nu^K)p_t K_t$. If $\Xi < 1$, households default even if they have some positive home equity.

Comparing the liquidity default cutoff in (73) to the optimal default threshold in (60), the key difference is that liquidity defaulters follow a heuristic default rule that reflects their need to alleviate a liquidity squeeze. Strategic default following (60) optimally trades off the shadow value of the house $Q_{K,t}$ against the present discounted value of outstanding liabilities, reflected by the shadow price $Q_{M,t}$ and $Q_{A,t}$.

General case with PO/IO and asymmetric indexation. We now derive the modified version of the borrower model that includes liquidity defaults in combination with PO/IO indexation and asymmetric indexation. These derivations nest the case of no or symmetric indexation.

Using the notation from Section 8.2, the general form of the liquidity default threshold is

$$\bar{\omega}_{i,t}^{U,Liq} = \frac{\mathcal{I}_M\left\{\min\left(\omega_{i,t}^{L}, \bar{\zeta}_\omega\omega_{i,t-1}^{L}\right)^{\iota_\omega}\right\}M_t^B + \mathcal{I}_A\left\{\min\left(\omega_{i,t}^{L}, \bar{\zeta}_\omega\omega_{i,t-1}^{L}\right)^{\iota_\omega}\right\}A_t^B}{(1-\Xi)\omega_{i,t}^{L}(1-\nu^K)p_t K_t^B}, \tag{74}$$

where $\mathcal{I}_j\{x\}$ is equal to x if $j = M, A$ is indexed, and is equal to 1 if that debt component is not indexed. If indexation is symmetric, then $\bar{\zeta}_\omega = \infty$ and the expression in (74) simplifies to (73).

Conditional on receiving a liquidity shock, we have (using the general $\omega_{i,t}$ notation from Section C.2):

$$Z_{D,t}^{Liq} = \int\int \Gamma_{\omega,t}^{U}\left(\bar{\omega}_{i,t}^{U,Liq}(\omega^L)\right)\, d\Gamma_{e,t}^{L}\, d\Gamma_{\omega,t-1}^{L}$$
$$Z_{N,t}^{Liq} = \int\int \Gamma_{\omega,t}^{U}\left(1-\bar{\omega}_{i,t}^{U,Liq}(\omega^L)\right)\, d\Gamma_{e,t}^{L}\, d\Gamma_{\omega,t-1}^{L}$$

$$
\begin{aligned}
Z_{K,t}^{Liq} &= \int\int \left(\int_{\bar{\omega}_{i,t}^{U,Liq}(\omega^L)} \omega_{i,t}^U \, d\Gamma_{\omega,t}^U\right) \omega_{i,t}^L \, d\Gamma_{e,t}^L \, d\Gamma_{\omega,t-1}^L \\
Z_{M,t}^{Liq} &= \int\int \Gamma_{\omega,t}^U \left(1 - \bar{\omega}_{i,t}^{U,Liq}(\omega^L)\right) \mathcal{I}_M(\omega^L)^{\iota_\omega} \, d\Gamma_{e,t}^L \, d\Gamma_{\omega,t-1}^L \\
Z_{A,t}^{Liq} &= \int\int \Gamma_{\omega,t}^U \left(1 - \bar{\omega}_{i,t}^{U,Liq}(\omega^L)\right) \mathcal{I}_A(\omega^L)^{\iota_\omega} \, d\Gamma_{e,t}^L \, d\Gamma_{\omega,t-1}^L.
\end{aligned}
$$

For strategic default, we introduce an extra cost to the borrower of losing his or her home, equal to η^B of the value of the home. This allows us to capture the observation that borrowers do not tend to strategically default until they are well under water. We rebate the cost from foreclosure lump-sum. The cost and its rebate appear as additional terms in the borrower's budget constraint:

$$
\begin{aligned}
C_t^B = {} & \underbrace{(1-\tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t}\left(Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B\right)}_{\text{net new borrowing}} - \underbrace{(1-\delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1-\tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\
& - \underbrace{p_t\left[Z_{R,t}Z_{N,t}K_t^* + \left(\nu^K - Z_{R,t}\right)Z_{K,t}K_t^B\right]}_{\text{owned housing}} - \underbrace{\rho_t\left(H_t^B - K_t^B\right)}_{\text{rental housing}} \\
& - \underbrace{(\Psi(Z_{R,t}) - \bar{\Psi}_t)Z_{N,t}M_t^*}_{\text{net transaction costs}} - \left(\underbrace{\eta^B(1 - Z_{K,t})p_t K_t^B - \text{Rebate}_t}_{\text{foreclosure costs}}\right) - \underbrace{T_t^B}_{\text{lump sum taxes}}.
\end{aligned} \tag{75}
$$

The strategic default threshold becomes:

$$
\bar{\omega}_{i,t}^{U,Str} = \frac{\mathcal{I}_M\left\{\min\left(\omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L\right)^{\iota_\omega}\right\} Q_{M,t}M_t^B + \mathcal{I}_A\left\{\min\left(\omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L\right)^{\iota_\omega}\right\} Q_{A,t}A_t^B}{(1+\eta^B)\omega_{i,t}^L Q_{K,t}K_t^B} \tag{76}
$$

where the Q terms are defined as above. Given this threshold, the corresponding Z^{Str} values can be computed by replacing Liq with Str above.

Total default rates are the weighted average of liquidity and strategic default rates with weights θ and $1-\theta$. More generally, the Z variables can now be computed as follows:

$$
Z_{l,t} = \theta Z_{l,t}^{Liq} + (1-\theta)Z_{l,t}^{Str},
$$

for $l = D, N, K, A, M$. This linear combination is valid if the liquidity default threshold is always strictly above the strategic default threshold, i.e., no households receiving liquidity shocks that do not liquidity-default would choose to strategically default. This assumption is always satisfied in our calibration.

Finally, we allow borrowers to internalize the effect of their housing and debt decisions on the future probability of liquidity default. The liquidity default threshold is mechanical, unlike the strategic default threshold which is optimally chosen. Thus, the envelope theorem does not apply, and the response of the liquidity default probability will enter the borrower's optimality conditions.

To aid notation, define $\Delta^{l}_{x,t} = \theta \partial Z^{Liq}_{M,t} / \partial x$ for a given variable x and superscript $l = N, K, M, A$, and define $\Delta^{BC}_{x,t}$ to be the derivative of the budget constraint with respect to x_t. Then we have:

$$\begin{aligned}
\Delta^{BC}_{x,t} &= Z_{R,t}\left(\Delta^{N}_{x,t} M^{*}_{t} - \delta \Delta^{M}_{x,t} M^{B}_{t}\right) - (1-\delta)\Delta^{M}_{x,t} M^{B}_{t} - (1-\tau)\Delta^{A}_{x,t} A^{B}_{t} \\
&\quad - p_t\left[Z_{R,t}\Delta^{N}_{x,t} K^{*}_{t} + (\nu^{K} - Z_{R,t})\Delta^{K}_{x,t} K^{B}_{t}\right] - \left(\Psi(Z_{R,t}) - \bar{\Psi}_t\right)\Delta^{N}_{x,t} M^{*}_{t} + \eta^{B}\Delta^{K}_{x,t} p_t K^{B}_{t}
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial Z^{Liq}_{D,t}}{\partial x} &= \int\int f^{U}_{\omega,t}\left(\bar{\omega}^{U,Liq}_{i,t}(\omega^{L})\right)\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial x}\, d\Gamma^{L}_{e,t}\, d\Gamma^{L}_{\omega,t-1} \\
\frac{\partial Z^{Liq}_{N,t}}{\partial x} &= -\int\int f^{U}_{\omega,t}\left(\bar{\omega}^{U,Liq}_{i,t}(\omega^{L})\right)\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial x}\, d\Gamma^{L}_{e,t}\, d\Gamma^{L}_{\omega,t-1} \\
\frac{\partial Z^{Liq}_{K,t}}{\partial x} &= -\int\int \bar{\omega}^{U,Liq}_{i,t}\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial x}\, d\Gamma^{U}_{\omega,t}\, \omega^{L}_{i,t}\, d\Gamma^{L}_{e,t}\, d\Gamma^{L}_{\omega,t-1} \\
\frac{\partial Z^{Liq}_{M,t}}{\partial x} &= -\int\int f^{U}_{\omega,t}\left(\bar{\omega}^{U,Liq}_{i,t}(\omega^{L})\right)\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial x}\mathcal{I}_{M}(\omega^{L})\, d\Gamma^{L}_{e,t}\, d\Gamma^{L}_{\omega,t-1} \\
\frac{\partial Z^{Liq}_{A,t}}{\partial x} &= -\int\int f^{U}_{\omega,t}\left(\bar{\omega}^{U,Liq}_{i,t}(\omega^{L})\right)\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial x}\mathcal{I}_{A}(\omega^{L})\, d\Gamma^{L}_{e,t}\, d\Gamma^{L}_{\omega,t-1}.
\end{aligned}$$

The derivatives of the threshold $\bar{\omega}^{U,Liq}_{i,t}$ with respect to the state variables are

$$\begin{aligned}
\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial K^{B}_{t}} &= -\frac{\bar{\omega}^{U,Liq}_{i,t}}{K^{B}_{t}} \\
\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial M^{B}_{t}} &= \frac{\mathcal{I}_{M}\left\{\min\left(\omega^{L}_{i,t}, \bar{\zeta}_{\omega}\omega^{L}_{i,t-1}\right)^{\iota_{\omega}}\right\}}{(1-\Xi)\omega^{L}_{i,t}(1-\nu^{K})p_t K^{B}_{t}} \\
\frac{\partial \bar{\omega}^{U,Liq}_{i,t}}{\partial A^{B}_{t}} &= \frac{\mathcal{I}_{A}\left\{\min\left(\omega^{L}_{i,t}, \bar{\zeta}_{\omega}\omega^{L}_{i,t-1}\right)^{\iota_{\omega}}\right\}}{(1-\Xi)\omega^{L}_{i,t}(1-\nu^{K})p_t K^{B}_{t}}.
\end{aligned}$$

The first-order condition for housing becomes:

$$p_t = \frac{\mathbb{E}_t\left\{\Lambda^{B}_{t+1}\left[\rho_t + \Delta^{BC}_{K,t+1} - (1-Z_{K,t+1})\eta^{B} p_{t+1} + Z_{K,t+1} p_{t+1}\left(1 - \nu^{K} - (1-Z_{R,t+1})\lambda^{LTV}_{t+1}\phi^{K}\right)\right]\right\}}{1 - \lambda^{LTV}_{t}\phi^{K}}.$$

The marginal continuation costs of debt for principal and interest become:

$$\begin{aligned}
\Omega_{M,t} &= \mathbb{E}_t\left\{\Lambda^{B}_{t+1}\bar{\pi}^{-1}\zeta_{p,t+1} Z_{M,t+1}\left[\Delta^{BC}_{M,t+1} + (1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1})\Omega_{M,t+1}\right]\right\}, \\
\Omega_{A,t} &= \mathbb{E}_t\left\{\Lambda^{B}_{t+1}\bar{\pi}^{-1}\zeta_{p,t+1} Z_{A,t+1}\left[\Delta^{BC}_{A,t} + (1-\tau) + \delta(1-Z_{R,t+1})\Omega_{A,t+1}\right]\right\}.
\end{aligned}$$

Calibration. As explained in Section 7, we calibrate the model with liquidity defaults to match total mortgage default rates while at the same time capturing that most households do not strategically default until they are well under water. By further setting $\Xi = 0.9$, we capture that some households default for liquidity reasons even though they have up to 10% positive home equity.

The model with liquidity defaults is qualitatively different from our baseline model in two aspects. First, the liquidity default threshold is mechanical, while the strategic default threshold optimally depends on the continuation costs $Q_{A,t}, Q_{M,t}, Q_{K,t}$ defined in equations (61) - (63). This means that borrowers internalize at time t that they may be driven into liquidity default at $t+1$, which causes precautionary effects in borrower demand for housing and debt. In particular, as we increase the fraction of liquidity shocks θ everything else equal, borrowers demand more housing and less debt to lower the future default threshold. To offset this effect and still hit our calibration target for borrower housing wealth to income and aggregate house price volatility, we reduce the utility parameter for housing to $[\xi_0, \xi_1] = [0.19, 0.13]$.

The fact that the default threshold is mechanical and we have set $\Xi = 0.9$ further means that exposing borrowers to liquidity default shocks also raises the overall default rate, since borrowers are defaulting more frequently than they would if default was strategic and without penalty ($\eta^B = 0$). Raising the strategic default cost η^B to 0.05 makes strategic default rare and helps to lower the overall default rate. However, to match our empirical targets, we also need to lower housing risk to $[\sigma_{\omega,0}, \sigma_{\omega,1}] = [0.16, 0.21]$.

The second important difference in the model with liquidity defaults follows logically from the first. Given that the re-calibrated model matches the same total default rate, but with a higher fraction of liquidity defaults, the overall loss rate of banks on their loan portfolio is lower. A strategically defaulting household optimally executes its default option, inflicting maximum losses on intermediaries. The liquidity-defaulting household, to the contrary, enters foreclosure with greater (possibly positive) home equity on average, since the default decision is triggered by a liquidity shock. We match the same REO discount as in the baseline model, implying a somewhat lower average loss-given-default in the model with liquidity defaults.

Results. As we explain in Section 7, liquidity defaults in our model are really "double-trigger" defaults, in the sense that both a liquidity shock and sufficiently low home equity are required to cause a foreclosure. Thus, a borrower's loan-to-value ratio is the key driver of both purely strategic and liquidity default, meaning that the optimal strategic threshold (60) co-moves closely with the mechanical liquidity threshold (73). With the exception of the two economies with IO indexation (Reg-IO and Asum-IO), the winners and losers in each policy experiment are the same, and the magnitudes of the gains and losses are similar.

The different effects of IO indexation in the models with and without liquidity defaults highlight how borrower default decisions vary across the two models. When interest payments become indexed, borrowers that receive a low idiosyncratic shock enjoy a permanent reduction in the interest rate they need to pay going forward (the state variable A_t^B is marked down). This is reflected in the strategic default threshold (76) through the shadow price $Q_{A,t}$ multiplying interest owed A_t^B. Liquidity defaulters, however, only consider interest payment relief this period as in (74) – the effective weight they assign to A_t^B is 1, and generally $Q_{A,t} >> 1$. Thus liquidity defaulting households do not optimally take into account the future interest payment relief resulting from indexation. As a result, the effects of IO indexation in the liquidity-default model are quantitatively small relative to no indexation. Further, by failing to take full advantage of future interest payment relief from indexation, borrowers effectively lose and intermediaries gain relative to no indexation. Hence in the liquidity-default model's Reg-IO and Asym-IO cases (Table 5), the welfare implications for borrowers and intermediaries are inverted relative to the strategic default model (Table 4).

C.4 Transition Path Results

The tables in the main text were steady-state comparisons. Table C.3 shows the change in variables in the first period of transition on the path between the "No Index" steady state, and the steady state of an alternative model. First-period value functions measure the total welfare change including the entire transition path to the new steady state.

C.5 Model with Government Debt

Figure C.1 shows impulse-responses in financial recessions with Aggregate indexation. It contrasts the model where all bailouts are finance through immediate taxation to the model where some of the bailout is paid for with debt.

Table C.1: Results: Intermediary Share, Regional Indexation

	No Index	Regional, Varying Intermediary Share 3%	4%	5% (Base)	6%	10%
			Borrower			
1. Housing capital	0.456	0.462	0.462	0.463	0.463	0.463
2. Refi rate	3.82%	3.69%	3.71%	3.73%	3.74%	3.75%
3. Default rate	0.97%	0.51%	0.50%	0.47%	0.47%	0.46%
4. Household leverage	64.31%	65.65%	65.62%	65.61%	65.73%	65.77%
5. Mortgage debt to income	250.06%	255.27%	258.78%	261.95%	267.51%	270.14%
6. Loss-given-default rate	37.31%	31.92%	33.63%	35.75%	36.62%	36.92%
7. Loss rate	0.40%	0.21%	0.20%	0.19%	0.19%	0.19%
			Intermediary			
8. Bank equity ratio	7.04%	7.21%	7.15%	7.22%	7.14%	7.07%
9. Bank default rate	0.30%	1.55%	1.24%	0.40%	0.27%	0.20%
10. DWL of bank defaults	0.03%	0.15%	0.12%	0.04%	0.03%	0.02%
11. Deposits	2.003	2.024	2.056	2.098	2.157	2.177
12. Saver mortgage share	14.43%	15.01%	14.83%	14.17%	13.74%	13.78%
			Prices			
13. House price	8.533	8.401	8.519	8.616	8.784	8.864
14. Risk-free rate	0.76%	0.71%	0.71%	0.77%	0.76%	0.76%
15. Mortgage rate	1.56%	1.51%	1.48%	1.43%	1.37%	1.34%
16. Credit spread	0.80%	0.80%	0.77%	0.66%	0.61%	0.59%
17. Mortgage risk prem.	0.40%	0.60%	0.57%	0.46%	0.42%	0.40%
			Welfare			
18. Aggregate welfare	0.872	-0.77%	-0.10%	+0.09%	+0.09%	+0.18%
19. Value function, B	0.398	-0.38%	-0.41%	+0.19%	+0.32%	+0.47%
20. Value function, S	0.408	+2.43%	+2.27%	+0.20%	+0.10%	-13.60%
21. Value function, I	0.066	-22.86%	-12.93%	-1.18%	-1.15%	+83.58%
			Consumption and Risk-sharing			
22. Consumption, B	0.382	-0.5%	-0.2%	+0.3%	+0.5%	+0.7%
23. Consumption, S	0.404	+4.9%	+2.2%	+0.2%	-2.8%	-13.4%
24. Consumption, I	0.067	-26.0%	-11.6%	-1.1%	+15.9%	+78.4%
25. Consumption gr vol, B	0.55%	+663.3%	+419.2%	+26.2%	-47.5%	-40.9%
26. Consumption gr vol, S	1.14%	-3.5%	-13.6%	-18.1%	-12.7%	-15.3%
27. Consumption gr vol, I	5.66%	+584.8%	+429.6%	+134.2%	+53.2%	-24.5%
28. Wealth gr vol, I	0.045	+1161.3%	+1098.8%	+375.9%	+185.5%	+65.0%
29. log (MU B / MU S) vol	0.026	+43.2%	+7.7%	-35.1%	-35.9%	-33.8%
30. log (MU B / MU I) vol	0.068	+260.0%	+186.7%	+66.1%	+28.3%	-25.3%

Table C.2: Transition Path Impacts

	No Index	Aggregate	Local	Regional
Welfare	0.872	-0.13%	+0.57%	+0.43%
Value function, B	0.398	-0.62%	+1.22%	+0.75%
Value function, S	0.408	+0.06%	+0.30%	+0.27%
Value function, I	0.066	+1.67%	-1.70%	-0.47%
Consumption, B	0.382	-1.24%	+2.94%	+1.34%
Consumption, S	0.404	+2.69%	-0.54%	+0.31%
Consumption, I	0.067	+4.38%	+0.17%	+4.35%

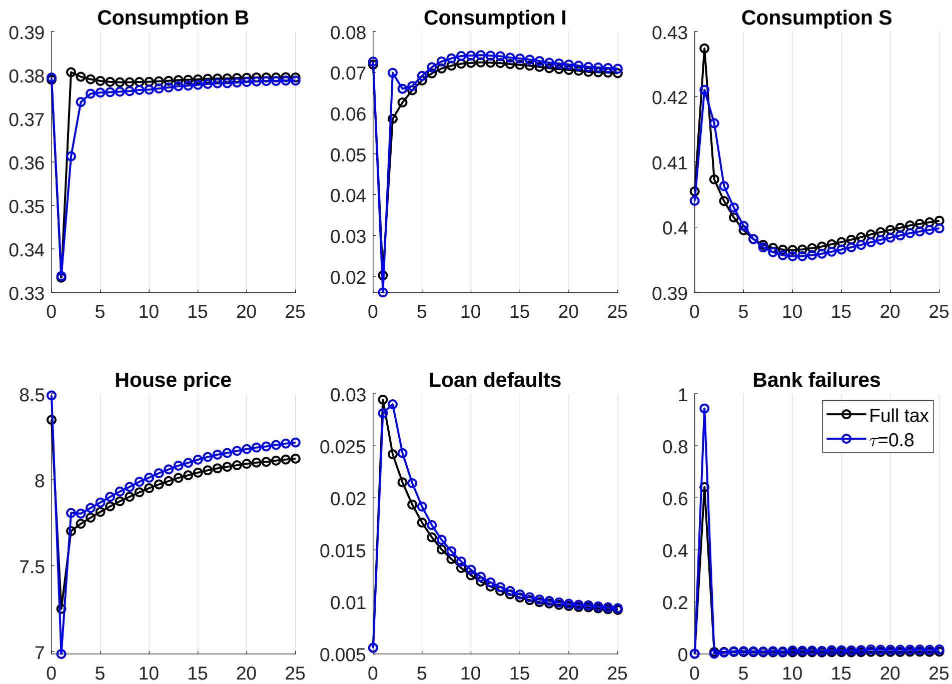
The table reports the initial change following a surprise switch from the baseline mortgage contract ("no index") to an alternative contract. Each transition path is computed from a random starting point simulated from the stationary distribution of the benchmark model. All flow variables are quarterly.

Table C.3: Transition Path Impacts (Alternative Indexation Schemes)

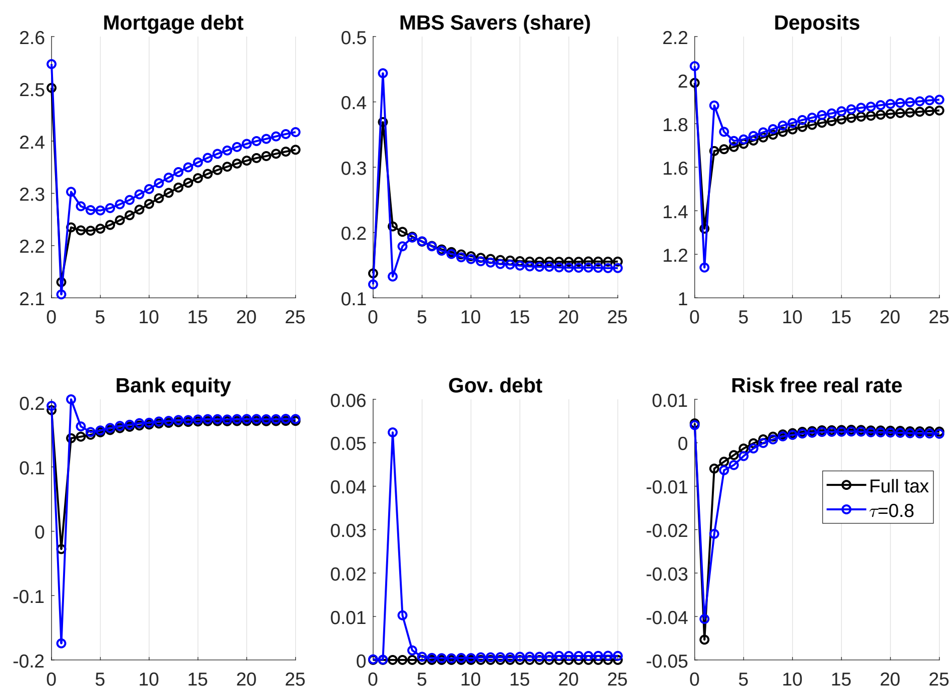
	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO	Reg-Tail
Welfare	0.872	+0.43%	+0.31%	+0.51%	+0.57%	+0.24%	+0.50%
Value function, B	0.398	+0.75%	+0.79%	+1.09%	+1.56%	+0.67%	+1.46%
Value function, S	0.408	+0.27%	+0.12%	+0.14%	-0.05%	+0.11%	+0.02%
Value function, I	0.066	-0.47%	-1.38%	-0.63%	-1.64%	-1.62%	-2.36%
Consumption, B	0.382	+1.34%	+2.18%	+2.24%	-0.96%	+1.10%	+0.18%
Consumption, S	0.404	+0.31%	+0.40%	-0.40%	+0.70%	+1.03%	+0.69%
Consumption, I	0.067	+4.35%	-1.37%	+3.35%	+15.21%	+1.07%	+8.55%

The table reports the initial change following a surprise switch from the baseline mortgage contract ("no index") to an alternative contract. Each transition path is computed from a random starting point simulated from the stationary distribution of the benchmark model. All flow variables are quarterly.

Figure C.1: Financial Recessions: Full vs. Partial Taxation for Bailout Funding



(a) Consumption and Financial Fragility



(b) Intermediary Sector and Government Debt

Black: financial recession in aggregate indexation baseline, **Blue:** financial recession with aggregate indexation and tax smoothing. Responses are plotted in levels.

D Online Appendix: Empirical Supplement

D.1 Empirical Evidence: Data and Additional Figures

This section describes additional detail for the data construction and regressions in Section 2 and provides supplementary plots.

Data Construction. Our house price data consist of three-digit ZIP code-level house price indices (All Transactions) from the Federal Housing Finance Agency. Our loan performance data come from Freddie Mac's Single Family Loan-Level Dataset, which provides detailed information on 25.9 million fixed-rate mortgages, including whether the loan went into delinquency or REO status (foreclosure), as well as the dollar loss that Freddie Mac took on the loan. We define delinquency as being 90+ days past due. To compute total loan losses to the lenders, we combine the total losses taken by Freddie Mac ("Actual Loss") with the losses to private mortgage insurers recovered by Freddie Mac ("MI Recoveries").

Regression Specifications. Our baseline regression for Figure 1a in the main text, as well as Figures D.1a and D.1c in this appendix, is specified by

$$y_{i,t\to t+20} = \phi_i + \sum_{j=1}^{N} \gamma_j \text{Bin}_{j,t\to t+20} + \varepsilon_{i,t\to t+20} \tag{77}$$

where i indexes the location (ZIP-3), ϕ_i is a location fixed effect, $y_{i,t\to t+20}$ is the outcome variable over the 20Q following origination, and $\text{Bin}_{j,t\to t+20}$ is a dummy for whether the local house price growth over the 20Q following origination lies in the jth bin, out of the 20 bins of width 5% that we define between -50% and 50% growth. We top and bottom code any observations outside our grid to fall in the final bins. For regression weights, we use total population for that ZIP-3 as measured in the 2000 census. The figures display the coefficient estimates of γ_j for each bin, along with standard errors that are double clustered at the ZIP-3 and quarter level.

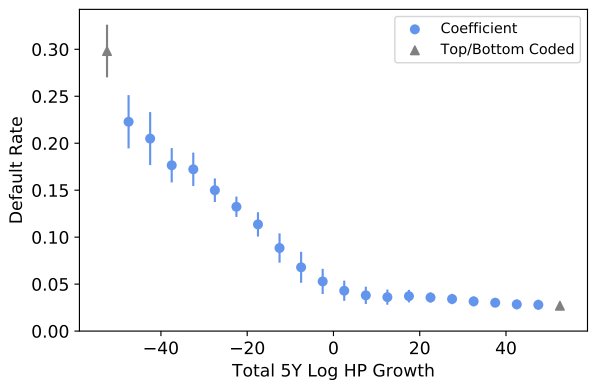
For our relative house price growth regression, plotted in Figure 1b in the main text, as well as Figures D.1b and D.1d in this appendix, we first compute the weighted average of 5-year log house price growth in each quarter, and subtract it from total 5-year log house price growth to obtain relative 5-year log house price growth, which we convert back to levels. We then run the regression

$$y_{i,t\to t+20} = \phi_i + \psi_t + \sum_{j=1}^{N} \gamma_j \text{Bin}_{j,t\to t+20} + \varepsilon_{i,t\to t+20} \tag{78}$$

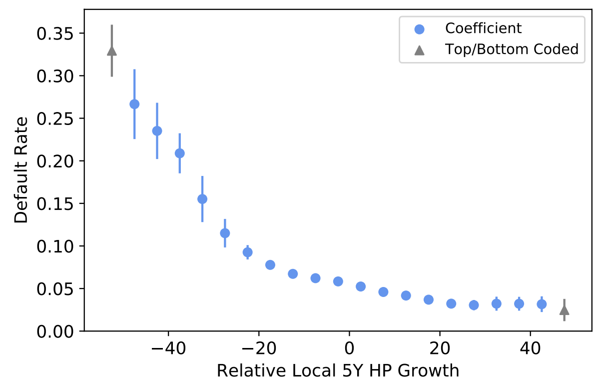
which is identical to (77), except for the inclusion of a time effect ψ_t to absorb any remaining influence of the national environment, and the redefinition of $\text{Bin}_{j,t\to t+20}$ to now bin over the relative house price growth measure just described, in place of the original bins over total house price growth. The figures again plot coefficient estimates of γ_j for each bin, along with standard errors that are double clustered at the ZIP-3 and quarter level.

Additional Figures D.1 display the coefficient estimates for the binned regression corresponding to Figures 1a and 1b in the main text, but using delinquency and REO status (foreclosure) as the dependent variables in place of loan losses. Similarly, D.2 display the plots corresponding to 1c and 1d in the main text using delinquency and REO status in place of loan losses.

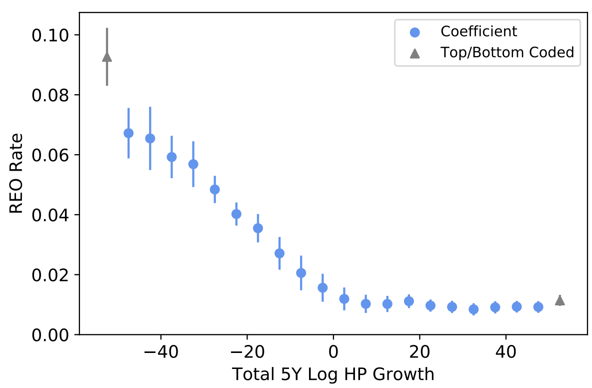
Figure D.1: Loan Defaults vs. House Prices



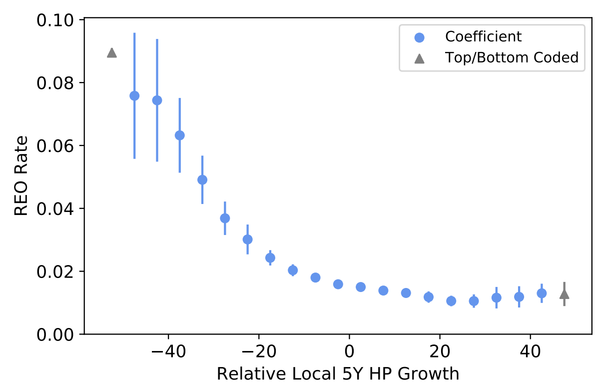
(a) Delinquent Rate by Total HP Growth



(b) Delinquent Rate by Local HP Growth



(c) REO Rate by Total HP Growth



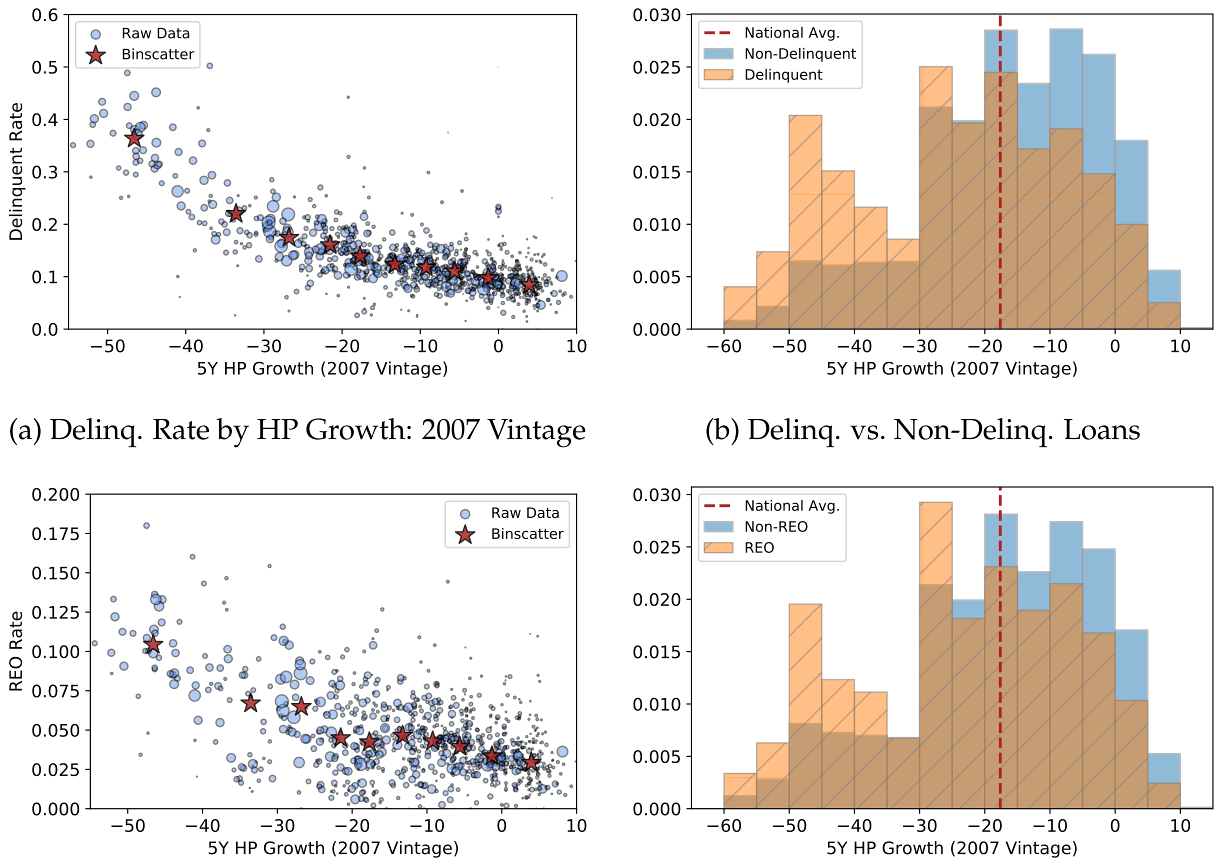
(d) REO Rate by Local HP Growth

Notes: Source is Freddie Mac Single Family Loan-Level Dataset.

D.2 Data Sources for Calibration

Aggregate Labor Income Labor income is defined as compensation of employees (line 2) plus proprietor's income (line 9) plus personal current transfer receipts (line 16) minus contributions to government social insurance (line 25), as given by Table 2.1 of the Bureau of Economic Analysis Na-

Figure D.2: Loan Defaults vs. House Prices: 2007 Vintage



(a) Delinq. Rate by HP Growth: 2007 Vintage

(b) Delinq. vs. Non-Delinq. Loans

(c) REO Rate by HP Growth: 2007 Vintage

(d) REO vs. Non-REO Loans

Notes: Source is Freddie Mac Single Family Loan-Level Dataset.

tional Income and Product Accounts. Deflation is by the personal income deflator and by population. Moments are computed in logs after removing a linear time trend.

Residential Mortgage Loans and default Rates Data are for all residential mortgage loans held by all U.S. banks, quarterly data from the New York Federal Reserve Bank from 1991.Q1 until 2016.Q4. The delinquency rate averages 2.28% per quarter between 2008.Q1 and 2013.Q4 (high uncertainty period, 23% of quarters) and 0.69% per quarter in the rest of the period.

Regional House Prices The data source is the Federal Housing Finance Agency All-Transactions House Price Index. The sample spans 1975.Q1 - 2017.Q1, and contains 13,649 observations drawn from 403 MSAs. The regression (37) is run using an unbalanced panel as MSAs enter the sample over time. The annual estimate is $\rho_{\omega}^{ann} = 0.911$ with standard error 0.004 (clustered at the MSA level). Results using a balanced panel limited to MSAs present since some given start date are nearly identical under a variety of start dates.

Bank Failures Based on the FDIC database of all bank failure and assistance transaction from 1991-2016, we calculate the asset-weighted average annual failure rate to be 1.65%.

Saver Mortgage Holdings We calibrate the two parameters of the cost function (level and elasticity to the quantity of holdings) by matching the mean and variance of the share of U.S. mortgages held outside the levered financial sector. Specifically, we use the Financial Accounts of the United States, Table L.211, on the holders of household mortgage debt. One very large category of "holder" are "mortgage pools and agency mortgage-backed securities." We use Table L.218 to split mortgage pools and agency MBS into their ultimate holders. We define one group of holders as levered financial institutions. This is the empirical counterpart to the model's intermediary sector. It consists of the categories: US deposit institutions, Foreign banking offices, banks in US-affiliated areas, credit unions, property and casualty insurance, life insurance companies, money market funds, brokers and dealers, holding companies, REITS, ABS issuers, GSEs, and the monetary authority. A second group consists of households and unlevered institutions. This group includes: households, private pension funds, federal government retirement funds, state and local government retirement funds, mutual funds. It is the empirical counterpart to our savers. We exclude a third group which is not in our model. It consists of the rest of the world, the non-financial corporate sector, the federal government, and state and local government. This group holds 11.5% of mortgage debt on average after 1991. We calculate the ratio of the holdings of the unlevered "saver" group to the sum of holdings of the saver and levered financial institutions group. We find an average saver share of mortgage debt of 15.0% over the 1991-2018 period. As an aside, this share is nearly unchanged (15.5%) if we reclassify the Fed's MBS holdings from the first (levered) to the third (excluded) group. The saver share is similar also if we start the time series in 1952 instead of 1991 (13.6%). The annual volatility of the saver share is 3.2%. We pick the two cost parameters for savers' holdings of mortgage debt to match the mean and volatility of saver holdings.